An information-based theory of financial intermediation*

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Abstract

We build a theory of financial intermediation based on the premise that some investors are better able to figure out the trade motives of their counterparties in bilateral meetings—screening experts. We solve for the equilibrium market structure, and study how information asymmetries stemming from heterogeneity in screening expertise shape up the core-periphery trade structure. In particular, the core of the market is populated by screening experts: they have the largest share of trade volume, they are actively engaged in middleman activity, and trade with the most counterparties. Using transaction-level micro data, and information disclosure requirements, we provide extensive evidence consistent only with a theory of financial intermediation building on screening expertise. Finally, we study the connection between expertise and efficiency: although higher expertise in the market translates into lower distortions in trade, higher expertise can also translate into a market with inefficient entry and intermediation levels.

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1 Introduction

Assets in over-the-counter (OTC) tend to be reallocated through a sequence of bilateral transactions that involve a core set of institutions serving as intermediaries.[1] Therefore, in order to understand how these markets function requires an understanding of the determinants of intermediation. In this paper, we build and empirically test a theory of intermediation based on a key friction inherent in decentralized trade: that market participants possess private information about their idiosyncratic willingness to pay for assets.

The theory predicts that if there is heterogeneity in the ability of market participants to learn the information of their counterparties, what we refer to as screening ability, then those investors that intermediate assets the most must have the highest screening ability. In other words, the core is endogenously comprised of experts—investors with the highest screening ability. We show this statement is true regardless of how intermediation and the core are defined; e.g. by an institution's share of trade volume, their balance of buy and sell orders, or their network connectedness to other market participants. We then provide empirical evidence, using transaction-level micro-data, to support our key predictions of the theory as well as illustrate that the interaction of private information and the incentive to intermediate assets is an important determinant of efficiency in decentralized trade.

The theory builds on the search-theoretic OTC market literature, Duffie, Garleanu, and Pedersen (2005) and Hugonnier, Lester, and Weill (2014), that model trade through random bilateral meetings between investors with heterogeneous private valuations of an asset. We augment the theory in two important ways. First, we assume that an investor's valuation of the flow of dividends is private information. While there is common knowledge about the dividend process, each investor is unaware of the private value of their counterparty. Traders can agree about the risk an asset pays off, but still do not know of each others' hedging, liquidity, or order-flow needs. While asymmetric information about common values has been a major focus of the literature, there is indirect evidence to suggest that private value uncertainty is a major detriment to trade in decentralized markets. For instance breakdowns in bilateral negotiations are common, even for assets with little common value uncertainty, as described in Merlo and Ortalo-Magne (2004) for residential houses, Backus, Blake, Larsen, and Tadelis (2020) for eBay products, and Larsen (2020) for wholesale business-to-business auto transactions.

Second, we assume that investors are heterogeneous in their ability to learn the private information of their trade counterparty, a technology we refer to as screening ability. Specifically, screening ability is the probability an investor learns the private information of their counterparty

[1] This is often referred to as a core-periphery market structure and has been widely documented in the literature, for instance, in the market for municipal bonds (Green, Hollifield, and Schurhoff, 2007), the Fed funds market (Bech and Atalay, 2010), the asset-backed securities market (Hollifield, Neklyudov, and Spatt, 2017), and the corporate bond market (Maggio, Kermani, and Song, 2016), among others.

in a meeting before trade takes place. This feature is meant to capture that institutions can differ according to their level of financial expertise, one aspect of which is having better information about the trading motives of other market participants.

We then show that intermediation is endogenously linked to screening ability. Our preferred measure of an investor's role in intermediation is the fraction of aggregate trade volume they account for, what we refer to as centrality. Investors with higher centrality intermediate assets the most and form the core of the market. We show that the most central investors are experts, those who possess the highest screening ability. We show that experts also tend to act as middlemen, buying and selling assets in proportion to the aggregate supply in the market. While there are also low-screening-ability investors that serve as middlemen, these investors always have lower trade volume. Finally, we show that the most central investors also have a larger trade network than the rest of the market resulting directly from their expertise.

To understand the intuition for why the investors most central in intermediating assets are experts, it is helpful to describe a few features of bilateral trade in partial equilibrium. Consider a meeting between a buyer and seller, and assume that one investor is randomly chosen to make a take-it-or-leave-it (TIOLI) offer in the form of a bid or ask price.[2] If the investor making the offer also observes the type of their counterparty —determined by their screening ability— then they extract all of the surplus in trade. However, if the investor is uninformed, as in Myerson (1981), they must set a distortive price that yields informational rents to their counterparty and destroys some efficient trades. In partial equilibrium, in the sense that we keep the investor's valuation for the asset constant, investors with higher screening ability are less likely to resort to setting distortive prices and so endogenously have a higher probability of trade. As a result, screening ability is a force that increases an investor's trading speed, increases their set of potential counterparties, and increases their expected profits from trade. In other words, in partial equilibrium, higher screening ability implies an investor possesses a better trading technology.

The intuition becomes more complicated in general equilibrium as an investor's screening ability affects their valuation, which in turn affects the bid and ask prices they face and the extent to which they intermediate assets. Indeed, in the aggregate the entire distribution of valuations affects optimal bid and ask prices, and vice-versa. This makes general equilibrium analysis significantly more cumbersome. However, we overcome these complications. We show an equilibrium exists by constructing a fixed point in the space of distributions and valuations. Our methodology is new to the literature and can be applied more broadly to environments that feature more complicated pricing mechanisms. In general equilibrium, it's possible that experts with too-low or too-high flow valuation for the asset may still overwhelmingly serve as sellers

[2] We show that TIOLI bid and ask prices are the solution to an optimal mechanism design problem that maximizes the respective profits of sellers and buyers in the meeting, which is an application of Myerson (1981). Additionally, we study mechanisms that maximize total surplus in the meetings and obtain the same qualitative results.

and buyers, respectively. However, our main results shows that the investors who intermediate assets the most, and form the core of the market, must still be experts.

In the second part of the paper, we provide empirical evidence that supports the primary result that heterogeneity in information leads to differences in intermediation activity. To do so, we use transaction-level data on the OTC market for credit-default-swap (CDS) indexes and examine the differential effects of information disclosure on an institution's trade with the core versus periphery. A subgroup of CDS-index traders in our sample are required to file a 13-F form to the Securities and Exchange Commission (SEC). The form contains the holdings of all securities regulated by the SEC, which mostly consist of equities that trade on an exchange and equity options. The SEC then makes the 13-F form public immediately after its filed and so other market participants know detailed portfolio information about 13-F filers. Since (i) CDS asset positions are small relative to the 13-F asset positions of these institutions and (ii) many institutions that file a 13-F do not trade CDS, we consider a 13-F filing as exogenous variation in the information the market possess about the institution's motives to trade CDS.[3]

We first extend the model to include a set of investors that file 13-F. We assume that filing a 13-F (imperfectly) reveals a filers' private information at a known future date, a shock that is independent of the screening ability of the filer's counterparties. The model predicts that a 13-F filing has heterogeneous effects on the filer's probability of trade with core versus periphery investors. Specifically, a 13-F filing discontinuously increases the probability of trade with the periphery, as once a filing occurs the periphery is more likely to know the filer's private valuation and are less likely to distort trade. However, a 13-F filing should have strictly less or no effect on trade with the core, as the model predicts that these investors already possess superior information. In other words, if the core is (at least in part) composed of institutions with better information, then information disclosure should effect these trades less than trades with the periphery.

We show these predictions hold in the CDS index market. We find that a 13-F filing increases an investor's probability of trade with the periphery in the week following a filing, but find either a zero or smaller increase in the probability of trade with the core. Our results are robust to controlling for different sets of fixed effects, classes of CDS indexes, and types of institutions. We also show that a 13-F filing only temporarily increases the probability of trade with the periphery up to two weeks after a filing, but the effect vanishes in week three and beyond. Further, we show the effect is quantitatively smaller in more liquid markets as measured by total trade volume. This result is consistent with our model that posits that private information about trade motives are less relevant in more competitive markets (i.e. markets with high contact rates). We conclude that these results are evidence that heterogeneity in information about private values is an important determinant of shaping the structure of OTC markets.

We also show our model is consistent with market-level heterogeneity in trade volume (cen-

[3]See Section 6.2 for a detailed discussion.

trality), middlemen activity, and network connections. As our model predicts, institutions that are more central according to their share of gross trade volume also serve as middlemen, buying and selling assets to lower volume institutions. While the market also consists of lower-volume middlemen, these institutions have lower screening ability as evidenced by a greater impact of 13-F filing on their trade activity compared to the high-volume middlemen. Further, institutions that are more central also have a more extensive trade network as measured by their share of counterparties. Our theory predicts this relationship as a direct result of heterogeneity in screening ability as the distortions caused by low-screening-ability institutions disrupt trade.

Finally, we explore the connection between screening ability and efficiency in OTC markets. We consider a version of the model with free entry, in which investors respond to the level of expertise in the market compared to their own screening ability. Even though more expertise is beneficial to investors in that it improves the reallocation of assets, it also reduces their competitive advantage by limiting informational rents. If this channel is strong enough, a market with no expertise —i.e. all investors are uninformed— may be welfare maximizing. We provide examples in which either everyone informed, no one informed, or an intermediate level of information is second-best. These results illustrate the effects of private information on the incentive to participate in OTC markets and intermediate trade. These incentives are central in understanding if there are benefits to increasing transparency in OTC markets.

The results in our paper follow a long tradition in economics of studying the role of information asymmetries in determining financial market outcomes. A recent literature, including Duffie, Malamud, and Manso (2009), Golosov, Lorenzoni, and Tsyvinski (2014), Guerrieri and Shimer (2014a), Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2015), Glode and Opp (2016), Malamud and Rostek (2017), and Babus and Kondor (2018), study information asymmetries in the context of decentralized asset markets. We differ from the majority of the papers in this literature in two ways. First, we consider asymmetric information about private values (information that only affects the individual payoff) as opposed to common values (information that affects the payoff of all agents in the economy).[4] Second, we focus on how intermediation arises endogenously, while previous work studying the interaction of information and intermediation, typically assume exogenous market makers.[5]

Our theory is consistent with other results in the literature about the determinants of core institutions. For instance, Üslü (2019) and Farboodi, Jarosch, and Shimer (2017) build theories where intermediaries are investors that possess a higher arrival rate of meetings, and thus trade

[4]Exceptions are Duffie (2012), Guerrieri and Shimer (2014b) and Chang (2017) that consider both private and common values and Guerrieri, Shimer, and Wright (2010), Cujean and Praz (2015), Zhang (2017) and Sultanum (2018) that consider private values, but do not study endogenous intermediation.

[5]An early example is Glosten and Milgrom (1985) who show, in the presence of adverse selection, market makers charge a positive bid-ask spread; a more recent example is Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2018) who study the role of market-makers in the presence of adverse selection and search frictions.

more often. Nosal, Wong, and Wright (2014) and Farboodi, Jarosch, and Menzio (2017) build a theory where intermediaries are investors that possess superior bargaining power, providing them with high profits from intermediation. In Chang and Zhang (2015) intermediaries are investors with low volatility in their flow valuation and, as a result, trade with a larger set of counterparties. We also find that intermediaries are investors with intermediate flow valuation, a result consistent with Hugonnier et al. (2014) and Afonso and Lagos (2015) in models similar to ours, as well as Atkeson, Eisfeldt, and Weill (2015) who show that intermediaries tend to have intermediate risk exposure.[6] A notable distinction between our environment and those mentioned, and one that is relevant for our empirical analysis, is that the key friction driving intermediation activity is the information structure.

2 Environment

Time is continuous and infinite and there is a measure one of infinitely-lived investors that discount the future at rate $r > 0$. There is transferable utility across investors and an endogenous supply of assets, s. Each unit of the asset pays a unit flow of dividends. The dividend flow is common knowledge and non-transferable—only the investor holding the asset consumes its dividends. Investors can hold either zero or one unit of the asset. We refer to investors holding an asset as owners, and to those not holding an asset as non-owners.

Trade occurs in a decentralized, over-the-counter market in the style of Duffie et al. (2005). Investors contact each other with Poisson arrival rate $\lambda/2 > 0$. Meetings between two investors that are owners result in no trade: agents can hold at most one asset and there are no gains from simply exchanging assets since they are the same. Likewise, meetings between two non-owner investors result in no trade. Only meetings between an owner investor and a non-owner investor can potentially involve gains from trade.

Investors are heterogeneous in two dimensions: they differ in their screening ability, α, and in the utility they derive from consuming the dividend flow of an asset, ν. When two investors meet, the screening ability α is public information, while the utility type ν is private information. The screening ability α determines the probability by which an investor learns the utility type of their counterparty. We let $\theta = (\alpha, \nu)$ denote the investor type. When two investors of types $\theta = (\alpha, \nu)$ and $\hat{\theta} = (\hat{\alpha}, \hat{\nu})$ meet, the investor with screening ability α learns $\hat{\nu}$ with probability α, and the investor with screening ability $\hat{\alpha}$ learns ν with probability $\hat{\alpha}$.[7]

[6]We are also related to the literature that model intermediation with explicit links between investors. Examples are Farboodi (2017), Babus and Kondor (2018) and Wang (2018).

[7]A natural question is whether one investor knows what the other investor knows in the meeting—that is, what information is common knowledge. Say two investors, A and B, meet, and investor A learns the utility type of investor B. Does investor B know that A knows his utility type? And does A know if B knows what he knows? To keep it simple, we assume that the information structure (who knows what) is common knowledge in a meeting. However, this assumption is without loss of generality. That is because whether an investor knows the utility type

Types are fixed over time, independent across investors, and are drawn from the cumulative distribution F. The distribution F has support $\Theta := \{\alpha^i\}_{i=1}^I \times \mathbb{R}$, which satisfies $0 \leq \alpha^1 < \alpha^2 < \ldots < \alpha^I = 1$, and, for each α^i, $F(\alpha^i, \nu) = \sum_{\alpha^i \leq \alpha} \int_{-\infty}^{\nu} f(\alpha^i, \tilde{\nu}) d\tilde{\nu}$ has a continuous density $f(\alpha^i, \cdot)$ and $\int \nu^2 F(\alpha^i, d\nu)$ is finite.[8] We assume that the screening ability α has finite support for technical reasons, but we can take the number of grid points to be as large as we want.

Assets mature and investors produce new assets following two Poisson distributions. An asset matures with Poisson arrival rate $\mu > 0$. When the asset matures, it disintegrates. With Poisson arrival rate $\eta > 0$, an investor faces an opportunity to issue a new asset at no cost. Investors can freely dispose of assets at any time.

As a result of maturity and issuance, a steady state with positive trade emerges in our economy even without time-varying types, which are required for existence of a steady state with trade in Duffie et al. (2005) and much of the literature that followed. Adding time-varying types in our setup is straightforward, but it does not provide additional insights so we do not incorporate this feature. Further, a convenient result of our approach is that, unlike in the model with time-varying types, the identity of core and periphery investors will be linked to the persistent type of the investor. This not only seems reasonable, in the sense we tend to believe that the identity of dealers, for example, does not vary through time, but will also allow us to study concretely the connection between an investor's type and her position in the trading network. This will be particularly relevant when trying to disentangle our model of intermediation building on information frictions from others in the literature.

3 Price setting, asset valuations, and allocations

In this section, we study price setting, we provide expressions for value functions and the distribution of assets among investors. We restrict attention to steady-state equilibria and omit the time t from the set of states of the economy. We denote the measures of owners and non-owners of type $\tilde{\theta} \leq \theta$ by $\Phi_o(\theta)$ and $\Phi_n(\theta)$, respectively, and the measure of assets by $s = \int d\Phi_o$. We denote the value function of an owner of type θ by $V_o(\theta)$ and the value function of a non-owner by $V_n(\theta)$. Finally, $\Delta(\theta)$ denotes the reservation value of an investor, which is the compensation that makes the investor indifferent between holding and not holding an asset.

The reservation value of an investor can be positive or negative because ν has support in the real numbers. A negative reservation value means the investor would have to be compensated to hold an asset. In this case, the investor would then dispose of the asset and, therefore, the

of the counterparty or not only informs the counterparty about the screening ability—there is no correlation between what an investor knows and his utility type. Since the screening ability of investors is public information, there is nothing to learn from whether an investor knows the utility type of the counterparty or not.

[8] We assume that ν has support in $\mathbb{R}$ to avoid dealing with corner solutions in the proofs. Because of the unbounded support, we assume that the distribution has a finite second moment, which we use in the equilibrium existence proof to guarantee that the optimal bid/ask prices exist and are finite.

equation $V_o(\theta) - V_n(\theta) = \max\{\Delta(\theta), 0\}$ must hold. This suggests an alternative interpretation of $\Delta(\theta)$: $\Delta(\theta)$ is the maximum gain an investor can get without disposing of the asset. If this value is negative, the investor is better off just disposing of the asset. Whenever it is not ambiguous, we use Δ_o instead of $\Delta(\theta_o)$, and Δ_n instead of $\Delta(\theta_n)$.

3.1 Bilateral trade

We cannot resort to Nash bargaining, or similar protocols, to determine the terms of trade due to private information in the meetings. Instead, we assume that when an owner and a non-owner meet, they play a random dictator game. With probability $\xi_o \in (0,1)$ the owner makes a take-it-or-leave-it offer, with commitment, to the non-owner that takes the form of an ask price. Likewise, with probability $\xi_n = 1 - \xi_o$ the non-owner makes a take-it-or-leave-it offer, with commitment, to the owner that takes the form of a bid price.

Two observations seem relevant regarding the random dictator game. First, the random dictator game allows us to accommodate different kinds of markets, regarding whether sellers or buyers typically set the terms of trade. For example, in the market for houses, typically the seller sets the terms of trade. While in the market for labor, the firm—that is, the renter/buyer of labor—offers a wage, thus setting the terms of trade. Second, while imposing bid and ask prices may seem restrictive, we show in Appendices A.2.1 and A.2.2 that this is equivalent to a generic mechanism design problem where the owner and non-owner maximize their respective, expected profits subject to individual rationality and incentive compatibility. That is, even when allowed to design complicated buying and selling mechanisms, take-it-or-leave-it bid and ask prices are indeed optimal when setting the terms of trade. Moreover, in the Online Appendix we explore a mechanism that maximizes the total trade surplus in a meeting, as in Myerson and Satterthwaite (1983), and obtain similar results.

The choice of ask and bid prices depend on whether or not the investor making an offer observes the utility type of their counterparty. When the investor making an offer observes the utility type of their counterparty, she can infer their reservation value and sets the terms of trade—the ask or bid price—to extract the entire gains from trade, if positive. That is, when the owner is informed, the optimal ask price is $ask_o = \Delta_n$ if $\Delta_o \leq \Delta_n$, and $ask_o = \Delta_o$ otherwise. Similarly, when the non-owner is informed, the optimal bid price is $bid_n = \Delta_o$ if $\Delta_o \leq \Delta_n$, and $bid_n = \Delta_n$ otherwise. Note that, in the particular case of the model with complete information, where $\alpha^1 = \alpha^I = 1$, the trade protocol replicates Nash bargaining with parameter ξ_o.

When the investor making an offer does not observe the utility type of their counterparty, they must set their ask and bid prices under private information. First consider the problem of the uninformed owner setting their ask price. The optimal ask price solves

$$\max_{ask} \; obj_o(ask; \alpha_n) := [ask - \Delta_o] \, [1 - M_n(ask; \alpha_n)] \, , \tag{1}$$

where α_n denotes the screening ability of the non-owner counterparty and

$$M_n(\tilde{\Delta};\alpha_n) = \frac{\int \mathbb{1}_{\{\Delta(\theta)\leq\tilde{\Delta},\ \alpha=\alpha_n\}} d\Phi_n(\theta)}{\int \mathbb{1}_{\{\alpha=\alpha_n\}} d\Phi_n(\theta)}$$

denotes the endogenous cumulative distribution of reservation values of non-owners with screening ability α_n. For a given ask, the measure $1 - M_n(ask;\alpha_n)$ of counterparties value the asset above the ask price and so accept the offer. If trade occurs, the owner receives the ask price and loses her reservation value for the asset Δ_o.

A solution to (1) exists if $M_n(\cdot;\alpha_n)$ has no mass points and finite second moments—which is satisfied in equilibrium. If problem (1) has multiple solutions, we let ask_o be the lowest ask price that solves (1). Similarly, when defining the optimal bid later in equation (2), we let it be the highest bid price that solves (2). However, we can show that the measure of investors that trade with positive probability and are indifferent between multiple bids or asks is zero in equilibrium, so the selection we use here is immaterial for our results.

The next lemma provides a useful result: the optimal ask price is strictly above the owner's reservation value whenever there are expected gains from trade. See Appendix A for all the proofs in the article.

Lemma 1. *Consider a meeting between an owner with reservation value Δ_o and a non-owner with screening ability α_n. Then, $1 - M_n(\Delta_o;\alpha_n) > 0$ implies that ask_o is strictly above Δ_o.*

It is easier to grasp the intuition for the above result when $M_n(\cdot;\alpha_n)$ is continuously differentiable. In this case, let $m_n(\cdot;\alpha_n)$ be the derivative of $M_n(\cdot;\alpha_n)$. The derivative of the objective function in (1) is then given by $\frac{\partial obj_o(\Delta_o,\alpha_n)}{\partial ask} = 1 - M_n(ask_o;\alpha_n) - [ask_o - \Delta_o]m_n(ask_o;\alpha_n)$. When evaluated at $ask_o = \Delta_o$, this derivative is positive whenever $1 - M_n(\Delta_o;\alpha_n) > 0$. That is because by increasing the ask price, at the margin, the owner gains an additional profit of $1 - M_n(\Delta_o;\alpha_n) > 0$. The increase in price prevent the owner from selling to non-owners with valuation exactly at the ask price, $m_n(ask_o;\alpha_n)$, but those sales do not generate profits when $ask_o = \Delta_o$. Therefore, when $1 - M_n(\Delta_o;\alpha_n) > 0$, the owner strictly prefers to set an ask price at a markup above their reservation value.

An implication from the previous discussion is that there exists a measure of non-owners, $M_n(ask_o;\alpha_n) - M_n(\Delta_o;\alpha_n)$, that have reservation value above the owner's reservation value but do not buy the asset. In other words, private information destroys efficient bilateral trades. This is a well-known, negative result from Myerson and Satterthwaite (1983) where the distributions, M_n and M_o, are exogenous. Here we show the result holds in a general equilibrium model of trade, but the same intuition applies. The following Corollary formalizes this result.

Corollary 1. *Consider a reservation value of owners Δ_o and a non-owner's screening ability α_n. If there exists $\bar{\epsilon} > 0$ such that $M_n(\Delta_o + \epsilon;\alpha_n) - M_n(\Delta_o;\alpha_n) > 0$ for all $\epsilon \in (0,\bar{\epsilon})$, then with positive* *probability the non-owner has a higher reservation value than the owner and they still do not trade—that is, $M_n(ask_o - \epsilon; \alpha_n) - M_n(\Delta_o; \alpha_n) > 0$ for some $\epsilon > 0$.*

The optimal bid price under private information follows closely optimal ask price discussed above, and is given by the solution to

$$\max_{bid} obj^n(bid; \alpha_o) := [\Delta_n - bid]\, M_o(bid; \alpha_o), \tag{2}$$

where

$$M_o(\tilde{\Delta}; \alpha_o) = \frac{\int \mathbb{1}_{\{\Delta(\theta) \leq \tilde{\Delta},\ \alpha = \alpha_o\}} d\Phi_o(\theta)}{\int \mathbb{1}_{\{\alpha = \alpha_o\}} d\Phi_o(\theta)}$$

denotes the endogenous cumulative distribution of reservation values of owners with screening ability α_o. For a given *bid*, the measure $M_o(bid; \alpha_o)$ of owners with screening ability α_o accept the offer and sell the asset to the non-owner. When the non-owner buys the asset, she gains her reservation value Δ_n and pays the bid price. As we discussed for the owner's problem, a solution to the buyer's problem presented in (2) exists as long as $M_o(\cdot; \alpha_o)$ satisfy some regularity condition, which we later verify in equilibrium.

Whereas the optimal ask price under private information is a markup over the reservation value of the owner, the opposite is true for the optimal bid price.

Lemma 2. *Consider a meeting between an non-owner with reservation value Δ_n and an owner with screening ability α_o. Then, $lim_{\Delta \nearrow \Delta_n} M_o(\Delta; \alpha_o) > 0$ implies that bid_n is strictly below Δ_n.*

The proof of Lemma 2 is analogous to the proof of Lemma 1 and thus we omit it here. Non-owners set a markdown under their reservation value when buying the asset. As a result, owners with reservation value below Δ_n and above the bid, will not sell the asset to the non-owner. Private information destroys bilaterally efficient trades, whether the owner or non-owner sets the terms of trade. We state this result below and omit its proof since it is analogous to the proof of Corollary 1.

Corollary 2. *Consider a reservation value of non-owners Δ_n and a owner's screening ability α_o. If there exists $\bar{\epsilon} > 0$ such that $M_o(\Delta_n; \alpha_o) - M_o(\Delta_n - \epsilon; \alpha_o) > 0$ for all $\epsilon \in (0, \bar{\epsilon})$, then with positive probability the non-owner has a higher reservation value than the owner and they still do not trade—that is, $M_o(\Delta_n - \epsilon; \alpha_o) - M_o(bid_n; \alpha_o) > 0$ for some $\epsilon > 0$.*

3.2 Expected gains from trade

The expected gains from trade in a meeting of an owner of type θ_o are given by

$$\begin{aligned}\pi_o(\theta_o) = \xi_o \int \alpha_o\, (\Delta_n - \Delta_o)\, \mathbb{1}_{\{\Delta_n \geq \Delta_o\}} + (1 - \alpha_o)\, (ask_o - \Delta_o)\, \mathbb{1}_{\{\Delta_n \geq ask_o\}} d\frac{\Phi_n(\theta_n)}{1 - s} \\ + \xi_n \int (1 - \alpha_n)\, (bid_n - \Delta_o)\, \mathbb{1}_{\{bid_n \geq \Delta_o\}} d\frac{\Phi_n(\theta_n)}{1 - s},\end{aligned} \tag{3}$$

and of a non-owner of type θ_n are given by

$$\pi_n(\theta_n) = \xi_n \int \alpha_n \left(\Delta_n - \Delta_o\right) \mathbb{1}_{\{\Delta_n \geq \Delta_o\}} + (1-\alpha_n)\left(\Delta_n - bid_n\right) \mathbb{1}_{\{bid_n \geq \Delta_o\}} d\frac{\Phi_o(\theta_o)}{s} + \xi_o \int (1-\alpha_o)\left(\Delta_n - ask_o\right) \mathbb{1}_{\{\Delta_n \geq ask_o\}} d\frac{\Phi_o(\theta_o)}{s}. \tag{4}$$

Consider equation (3). The first term accounts for the expected profits when the owner is selected to make the offer, which occurs with probability ξ_o. In this case, with probability α_o the owner is informed about the utility type of her counterparty and uninformed otherwise. When she is informed, the owner trades with any non-owner with reservation value larger than Δ_o and receives the entire trade surplus, $\Delta_n - \Delta_o$. When uninformed, the owner sets an ask price under private information and gets expected profits according to (1). The second term accounts for the expected profits when the non-owner is selected to make the offer, which occurs with probability ξ_n. In this case, the owner only receives positive profits if (i) her trade counterparty is uninformed, which occurs with probability $1-\alpha_n$, and (ii) her reservation value is below the optimal bid price of the non-owner. Otherwise, whenever the non-owner is informed about the utility type of the owner, the non-owner extracts the entire gains from trade. Finally, notice that the owner takes expectations over the endogenous distribution of non-owners, $\Phi_n(\theta_n)/(1-s)$. The expected gains from trade in a meeting of a non-owner of type θ_n, presented in equation (4), follow analogously to equation (3).

3.3 Value functions and reservation value

The value function of an owner of a type θ satisfies

$$rV_o(\theta) = \max\left\{\nu - \mu\left[V_o(\theta) - V_n(\theta)\right] + \lambda(1-s)\pi_o(\theta), rV_n(\theta)\right\}. \tag{5}$$

The value function of an owner of a type θ, discounted at rate r, is the maximum between the value of owning an asset and the value of disposing of it. The value of owning an asset equals the sum of three terms. The first term accounts for the flow utility of holding the asset, ν. The second term accounts for the change in value when the asset matures and the owner becomes a non-owner, which occurs at rate μ. The third term accounts for the expected profits of an owner when meeting a non-owner; the probability that two investors meet is given by $2\lambda/2 = \lambda$ and, conditional on meeting, the owner contacts a non-owner with probability $(1-s)$. Finally, the value of disposing of an asset equals the value of being an non-owner.

The value function of a non-owner of a type θ satisfies

$$rV_n(\theta) = \eta\left[V_o(\theta) - V_n(\theta)\right] + \lambda s \pi_n(\theta). \tag{6}$$

The value of not owning an asset, discounted at rate r, equals the sum of two terms. The first term accounts for the value of receiving an issuance opportunity, which arrives at rate η. Conditional on receiving an issuance opportunity, the non-owner decides whether it is optimal to produce the asset and become an owner, or to not produce and remain a non-owner. The non-owner can always issue the asset and then dispose of it, so $V_o(\theta) \geq V_n(\theta)$, as we can see from equation (5). As a result, we can without loss of generality assume that the asset is always issued when an issuance opportunity arrives. The second term accounts for the expected profits of a non-owner in bilateral trade, where a meeting occurs with an owner at rate λs.

Finally, the reservation value for an investor of type θ, $\Delta(\theta)$, satisfies

$$\Delta(\theta) = \underbrace{\frac{\nu}{r+\mu+\eta}}_{\substack{\text{fundamental}\\\text{value}}} + \underbrace{\frac{\lambda(1-s)\pi_o(\theta)}{r+\mu+\eta}}_{\substack{\text{option value}\\\text{to sell}}} - \underbrace{\frac{\lambda s \pi_n(\theta)}{r+\mu+\eta}}_{\substack{\text{option value}\\\text{to buy}}}. \tag{7}$$

Equation (7) decomposes the reservation value into three components. The first component represents the fundamental value of holding an asset, the discounted utility of the dividend payoff. The second and third components represent the option values of selling and buying the asset, respectively. What is important for the reservation value is the net, or the expected gain in value from buying then selling an asset, or the expected gain from intermediation.

By manipulating equations (5) and (6), we can show that $V_o(\theta) - V_n(\theta) = \max\{\Delta(\theta), 0\}$ for all θ. That is, whenever it is profitable for an investor to hold an asset until selling it, the reservation value is exactly the difference between the value of owning and not owning an asset. If θ is such that $\Delta(\theta) < 0$, on the other hand, we must have that $V_o(\theta) = V_n(\theta)$ since an owner would immediately dispose of any asset in her possession. Moreover, such investor would never hold any asset and we must have that the value of owning and not owning an asset both equal zero.

3.4 The distribution of assets

For each θ, $f(\theta)$ represents the density of investors with utility type θ. These investors can be owners, with density $\phi_o(\theta)$, and non-owners, which have density $\phi_n(\theta)$, and satisfy $\phi_o(\theta) + \phi_n(\theta) = f(\theta)$. Investors with negative reservation value immediately dispose of any asset. As a result, when θ is such that $\Delta(\theta) < 0$, we have that $\dot{\phi}_o(\theta) = \phi_o(\theta) = 0$ and $\phi_n(\theta) = f(\theta)$. When θ is such that $\Delta(\theta) \geq 0$, the change over time in the measure of owners of type θ satisfies

$$\begin{aligned}\dot{\phi}_o(\theta) &= \eta\phi_n(\theta) - \mu\phi_o(\theta) - \lambda\phi_o(\theta)\bar{q}_o(\theta) + \lambda\phi_n(\theta)\bar{q}_n(\theta) \\ &= \eta[f(\theta) - \phi_o(\theta)] - \mu\phi_o(\theta) - \lambda\phi_o(\theta)\bar{q}_o(\theta) + \lambda[f(\theta) - \phi_o(\theta)]\bar{q}_n(\theta) = 0,\end{aligned} \tag{8}$$

where the measure of meetings that an owner of type θ sells an asset is

$$\bar{q}_o(\theta) = \int q(\theta, \theta_n) d\Phi_n(\theta_n), \tag{9}$$

the measure of meetings that a non-owner of type θ buys an asset is

$$\bar{q}_n(\theta) = \int q(\theta_o, \theta) d\Phi_o(\theta_o), \tag{10}$$

and the probability that an owner of type θ_o sells an asset to a non-owner of type θ_n is

$$q(\theta_o, \theta_n) = \mathbb{1}_{\{\Delta_n \geq \Delta_o\}} - \xi_o(1-\alpha_o)\mathbb{1}_{\{ask_o > \Delta_n \geq \Delta_o\}} - \xi_n(1-\alpha_n)\mathbb{1}_{\{\Delta_n \geq \Delta_o > bid_n\}}. \tag{11}$$

The first term on the right-hand side of (8) accounts for the inflow of non-owners of type θ that become owners because they receive an issuance opportunity and find it worthwhile to produce the asset. The second term accounts for the outflow of owners of type θ because of asset maturity. The third term accounts for the outflow of owners of type θ that sell their asset. The fourth term accounts for the inflow of non-owners of type θ that buy an asset. A steady-state equilibrium satisfies $\dot{\phi}_o(\theta) = 0$ for all θ.

Given the density of owners, ϕ_o, the measure of owners then satisfies

$$\Phi_o(\theta) = \int_{\tilde{\theta} \leq \theta} \phi_o(\tilde{\theta}) d\tilde{\theta} = \sum_{\tilde{\alpha} \in \{\alpha^i\}_{i=1}^I} \mathbb{1}_{\{\tilde{\alpha} \leq \alpha\}} \int_{-\infty}^{\nu} \phi_o(\tilde{\theta}) d\tilde{\nu}. \tag{12}$$

Note that, because α is discrete, the integral over θ is defined as the sum over α and integral over ν. We use this notation in the remaining of the paper unless it is ambiguous. Since the measure of investors, F, is exogenous, we can obtain an expression for the measure of non-owners from the following equilibrium condition,

$$\Phi_o(\theta) + \Phi_n(\theta) = F(\theta). \tag{13}$$

Finally, all assets in the economy are held by owners so the stock of assets is

$$s = \lim_{\nu \nearrow \infty} \Phi_o(\alpha_I, \nu). \tag{14}$$

4 Equilibrium

We focus on symmetric steady-state equilibria.

Definition 1. *A family $\{bid_n, ask_o, \Delta, \Phi_o, \Phi_n, s\}$ constitutes a symmetric steady-state equilibrium if it satisfies: (i) the ask function, ask_o, solves the owner's problem* (1)*, and the bid function, bid_n, solves the non-owner's problem* (2)*; (ii) the reservation value, Δ, satisfies* (7)*, where π_o and π_n are given by* (3) *and* (4)*; and (iii) the distribution of owners, Φ_o satisfies* (12) *with ϕ_o satisfying* (8)*, the distribution of non-owners, Φ_n, satisfies* (13)*, and the stock of assets, s, satisfies equation* (14).

Note that the equilibrium definition does not include the value functions V_o and V_n because we can recover them from equations (5) and (6).

The family that constitutes an equilibrium can be understood as containing two sets. The first set contains the bid and ask functions, (bid_n, ask_o), and provides trade probabilities and asset prices. The second set contains the reservation value function and the asset distribution across types, defined by (Δ, Φ_o).[9] This set provides the distribution of reservation values and asset

[9]The asset distribution of non-owners and the total asset supply can be obtained from the identities $\Phi_o + \Phi_n = F$

holdings. The two components depend on each other. The bid and ask prices, by providing the trade probabilities, determine the asset distribution across types. Asset prices, combined with the asset distribution across types, determine reservation values. Reservation values and the asset distribution across types determines optimal bid and ask prices, as shown in Section 3.1. Because the sets are intertwined, an equilibrium is essentially a fixed point on these equilibrium objects.

The complexity arising from the connection between trade outcomes and reservation values and asset distribution across types is a feature not only of our model, but rather a general characteristic of the search-theoretic OTC literature that builds on Duffie et al. (2005). However, our approach to dealing with this complexity is quite different from the literature. If trade occurs under complete information, as assumed in most of the literature, we can easily characterize the connection between trade outcomes, reservation values and asset distribution across types.

To see this, notice Duffie et al. (2005), Hugonnier et al. (2014), Farboodi, Jarosch, and Shimer (2017) and Farboodi, Jarosch, and Menzio (2017) all study OTC trading under complete information, and provide a constructive proof of equilibrium existence.[10] Relative to our setup with private information, under complete information investors do not need to know the asset distribution across types when setting their bid and ask strategies. Further, since reservation values Δ are increasing in the flow valuation of investors ν for any distribution of types, trade occurs whenever $\nu_n \geq \nu_o$. These two observations provide that the trade pattern is resolved without knowledge of the reservation values or asset distribution. With the trade pattern at hand, it is straightforward to solve for the asset distribution across types and in turn one can recover reservation values.

In our environment under private information this approach is not feasible: trade outcomes depend on reservation values and the asset distribution across types, as well do optimal prices.[11] This prevents us from a constructive existence proof so instead we rely on a fixed point theorem. An advantage of our approach is that it is quite general: it can be applied to existing models with complete information and, as long as the trading protocol preserves some form of continuity, it can be applied to any other model that builds on Duffie et al. (2005), with or without trading frictions. These can include models in which pricing is determined by a generic mechanism.

The next Proposition guarantees that an equilibrium exists in the economy and, further, that imposing trade using bid and ask prices is not restrictive.

Proposition 1. *There exists a symmetric steady-state equilibrium, with bid and ask prices associated with optimal buying and selling mechanisms.*

and $s = \lim_{\nu \nearrow \infty} \Phi_o(\alpha_I, \nu)$.

[10] Actually, in most cases, they are all able to provide an exhaustive characterization of the equilibrium.

[11] Another approach is that one in Üslü (2019) where the insight is that, under an appropriate choice of the utility function and assumptions regarding the exogenous distributions of types, under complete information the trade outcomes only depend on the first moment of the asset distribution. Thus, the fixed-point can be reduced to a set of functional equations that can then be studied using standard techniques. This approach is not feasible in our setting, due to private information: the optimal trade mechanism requires more of the asset distribution than its first moment.

To prove existence we exploit the fact that bid and ask prices are intertwined with reservation values and the asset distribution across types. Given a pair (Δ, Φ_o) of reservation values and the asset distribution of owners, we obtain bid and ask functions (bid_n, ask_o). The bid and ask functions generate trade probabilities and asset prices that we use to update the initial guess (Δ, Φ_o) and obtain the operator $T(\Delta, \Phi_o) = (\hat{\Delta}, \hat{\Phi}_o)$. An equilibrium is characterized by a pair of reservation values and asset distribution of owners, (Δ, Φ_o), satisfying $T(\Delta, \Phi_o) = (\Delta, \Phi_o)$.

We apply the Schauder fixed point theorem to the operator T to show that it has a fixed point. However, it is not trivial to show that the conditions necessary to apply the fixed point theorem hold in the model. Two conditions are particularly difficult: the operator has to map a compact set into itself, and the operator has to be continuous. In what follows, we describe the intuition of the problem and how we overcome it. Readers less interested in these more technical details can freely skip to Section 5 without any loss.

Although the functions (Δ, Φ_o) are continuous, the space of continuous functions—even if uniformly bounded and defined on a compact support—is not compact. We deal with this by working in a set of Lipschitz continuous functions with the same constant.[12] In this set, the Arzelà-Ascoli theorem implies that any sequence has a converging sub-sequence, provided that the set is compact. For the theorem to work, however, we also need to show that T preserves Lipschitz continuity, guaranteeing that T maps the set into itself. We derive this result from the following observation. If the change in the reservation value Δ implied by a change from ν_1 to any finite $\nu_2 > \nu_1$ is bounded by $\frac{\nu_2-\nu_1}{\lambda+r+\mu+\eta} \leq \Delta(\alpha, \nu_2) - \Delta(\alpha, \nu_1) \leq \frac{\nu_2-\nu_1}{r+\mu+\eta}$, then $\hat{\Delta}$ satisfies the same property. We can similarly bound the variations in Φ_o using

$$\frac{\eta}{\lambda+\mu+\eta} \inf_{\nu \in [\nu_1, \nu_2]} \{f(\alpha, \nu)\} \quad \text{and} \quad \frac{\lambda+\eta}{\lambda+\mu+\eta} \sup_{\nu \in [\nu_1, \nu_2]} \{f(\alpha, \nu)\},$$

to obtain the uniform Lipschitz constant.[13] Once we have established the compactness of the domain of T, we need to show that T is continuous on (Δ, Φ_o). Showing continuity is particularly hard in our setting due to private information. Most of the literature building on complete information focuses on Nash bargaining. With Nash bargaining, trade occurs in a meeting when the reservation value of non-owners is above the reservation of owners, and the price is a weighted average of the reservation values. In this case, it is easy to see that small changes in the reservation value translate into small changes in trade probabilities and prices. Further, the asset distribution across types does not show up in the bargaining outcomes—except through its in-

[12]A function $f : X \to Y$ between metric spaces (X, d_X) and (Y, d_Y) is Lipschitz continuous if there exists a $K \geq 0$ such that $d_Y(f(x_1), f(x_2)) \leq K d_X(x_1, x_2)$ for all $x_1, x_2 \in X$. K is called the Lipschitz constant.

[13]The bounds we obtain for the variation of Δ and Φ_o are not without economic meaning. For example, the increase in reservation value Δ implied by a change from ν_1 to ν_2 cannot be larger than the variation we would observe when the trade probability is zero. The reason is that an increase in ν reduces an investor's profits when selling. Although it increases the profits when buying an asset, this decreases the reservation value because it increases the value of being an non-owner. Both these variations contribute negatively to the change in reservation value so we obtain an upper bound when the trade probability is zero. The other bound carries similar economic intuition.

direct effect on the reservation value. Therefore, small changes in the asset distribution across types also cannot generate jumps in trade probabilities or prices. As a result, in models building on complete information using Nash bargaining, continuity is easily guaranteed.

In our setting things are different. For example, an owner deciding on an ask price is trading off selling at a high price infrequently with selling at a low price frequently. It can happen that these two forces cancel each other, generating multiple optimal ask prices. A sufficient condition for the ask to be well behaved is that the hazard ratio of the distribution of non-owners is monotone. However, the distribution is endogenous so we cannot impose that to be the case. When the hazard ratio is not monotone and multiple optimal ask prices arise, small changes in the reservation value function Δ or the asset distribution distribution Φ_o can make one of the ask prices to be strictly preferred by the owner, causing a jump in trade probabilities. This could prevent the operator T from being continuous.

We show T is continuous in spite of the argument above. The reason is that jumps in trade probabilities occur in a set with zero measure. Therefore, although they can manifest on the densities of Φ_o and Φ_n, the uniform convergence of the cumulative distributions is preserved. Further, because these jumps occur when investors that want to sell an asset are indifferent between multiple ask prices, the expected gains from trade do not jump. An identical argument follows for investors that want to buy an asset, and are designing optimal bid prices. As a result, we can also show uniform convergence of reservation values.

A useful result we will use later is that reservation values are increasing in the utility type, ν.

Lemma 3. *In any symmetric steady-state equilibrium $\{bid_n, ask_o, \Delta, \Phi_o, \Phi_n, s\}$, the reservation value $\Delta(\alpha, \cdot)$ is continuous and strictly increasing in ν for every α. Moreover, $\lim_{\nu \nearrow \infty} \Delta(\alpha, \nu) = \infty$ and $\lim_{\nu \searrow -\infty} \Delta(\alpha, \nu) = -\infty$ hold for any α.*

5 Private Information and Intermediation

We now study how private information shapes an investor's role in intermediation, or the process by which assets flow from low-value investors to high-value investors. We consider several measures that capture an investor's role in re-allocating assets, including their share in gross trade volume (Section 5.1), their propensity to serve both sides of the market as buyer and seller (Section 5.2), and their degree of connectedness to other investors (Section 5.3). We also study the way information influences rents in the trading network (Section 5.4).

5.1 Screening Ability and Centrality

We start by defining an investor's centrality as the share of aggregate trade volume they account for. Given the measures of investors and trading probabilities for owners and non-owners, $\phi_o(\theta)$,

$\phi_n(\theta)$, $\bar{q}_o(\theta)$ and $\bar{q}_n(\theta)$, associated with equations (8)-(10), centrality is given by

$$c(\theta) = \frac{\lambda}{2Vol} \times \frac{\phi_o(\theta)\bar{q}_o(\theta) + \phi_n(\theta)\bar{q}_n(\theta)}{f(\theta)}, \tag{15}$$

where $Vol = \lambda \int \int q(\theta_o, \theta_n) d\Phi_o(\theta_o) d\Phi_n(\theta_n)$ is total trade volume and $\int c(\theta) f(\theta) d\theta = 1$. In equation (15), the expected probabilities of trade are weighted by the fraction of time spent as buyer or seller, ϕ_n/f and ϕ_o/f, respectively. In order to have high centrality an investor must not only have a high probability of trade conditional on a meeting but also have a high rate of meetings with gains from trade. For instance, the investor with the lowest asset valuation will have a high probability of selling an asset, since it is likely that any non-owner they meet will value the asset more than her, but they will spend little time possessing an asset in equilibrium and so seldom get the chance to sell. Further, the definition of centrality suggests that high centrality could be associated with a high trade frequency as both buyer and seller or, in other words, with high middleman activity. Section 5.2 studies in detail this connection.

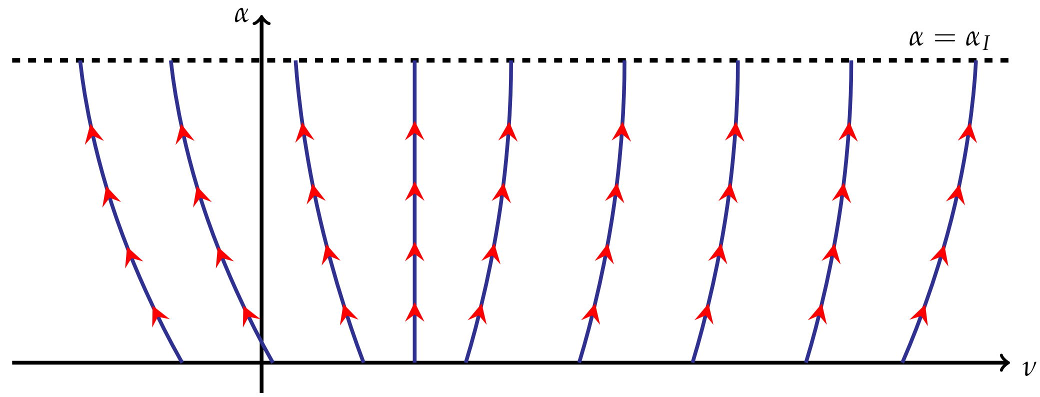
The following lemma establishes a useful partial equilibrium result: if two investors have the same reservation value for an asset, the investor with higher screening ability is more central.

Lemma 4. *Consider a symmetric steady-state equilibrium $\{bid_n, ask_o, \Delta, \Phi_o, \Phi_n, s\}$, and let the types $\theta = (\alpha, \nu)$ and $\hat{\theta} = (\hat{\alpha}, \hat{\nu})$ satisfy $\Delta(\theta) = \Delta(\hat{\theta})$ and $\alpha > \hat{\alpha}$. Then, (i) if $\Delta(\theta) = \Delta(\hat{\theta}) < 0$, we have that $c(\theta) = c(\hat{\theta}) = 0$, and (ii) if $\Delta(\theta) = \Delta(\hat{\theta}) \geq 0$, we have that $c(\theta) > c(\hat{\theta}) > 0$.*

Figure 1 illustrates the result in Lemma 4. It shows the level curves of the reservation value in a two-dimensional graph with the utility type ν on the horizontal axis and screening ability α on the vertical axis. For a given level curve, the red arrows indicate the direction in which centrality increases. Conditional on having the same reservation value, an investor with a higher screening ability will have a higher probability of trade in a meeting—as either a buyer or seller—because better information leads to a better trading technology in equilibrium. Lemma 20 in the Appendix formalizes this result. It then follows that, for a given reservation value, higher screening ability α implies higher centrality.

The reservation value level curves can be upward or downward sloping, as we illustrate in Figure 1. The reservation value is always increasing in utility type ν because it represents the flow value of holding an asset. However, the effect of increasing screening ability α depends on its effect on the difference between the option value of selling relative to buying, as can be seen in equation (7). When ν is high, investors are typically buyers in equilibrium as it is very costly for them to give up the utility stream that follows from holding an asset. As a result, screening ability affects their option value of buying more relative to selling since they seldom meet investors who value the asset more but often meet investors who value it less. Hence, in order to keep the reservation value constant as ν increases, the option value of buying must also increase (lowering the reservation value). This requires a higher screening ability α. When ν is

Figure 1: Reservation value level curves



Notes: The figure presents reservation value level curves –i.e. $\Delta(\theta) = \tilde{\Delta}$– as a function of asset valuation ν and screening ability α. Each blue line represents a different level curve. The red arrows represent the direction by which centrality increases, for a given reservation value level curve.

low, the opposite is true and α must decrease to keep the reservation value constant.

We now turn to studying how information shapes centrality in general equilibrium.

Definition 1. *An investor type θ^* is the most central if $c(\theta^*) \geq c(\theta)$ for all $\theta \in \Theta$.*

Proposition 2. *Consider a symmetric steady-state equilibrium $\{bid_n, ask_o, \Delta, \Phi_o, \Phi_n, s\}$. If an investor type $\theta^* = (\alpha^*, \nu^*)$ is the most central, then $\alpha^* = a_I$ and $c(\theta^*) > c(\theta)$ for all investors type $\theta \in \Theta$ that are not screening experts (that is, satisfying $\alpha < a_I$).*

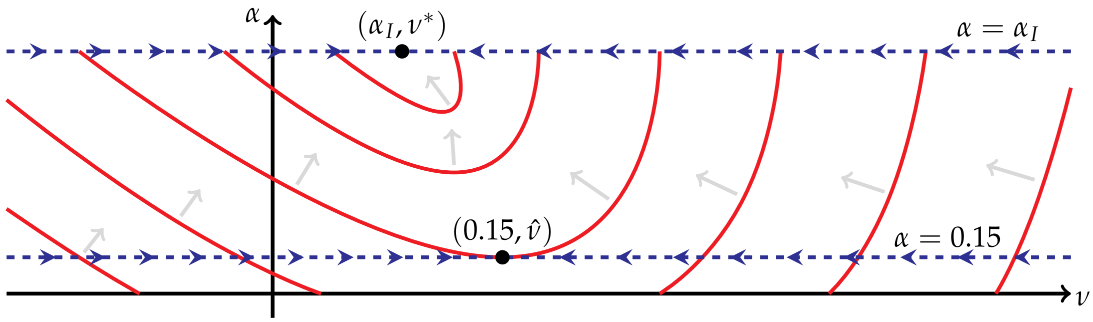
Figure 2 illustrates the results in Proposition 2. It shows the level curves of centrality in a two-dimensional graph, with screening ability α and utility type ν. The investors most central in intermediating assets must have the highest screening ability. To understand the intuition for this result, consider any investor $\theta = (\nu, \alpha)$ with $\alpha < \alpha_I$. We can show that $\lim_{\nu \searrow -\infty} \Delta(\alpha_I, \nu) = -\infty$ and $\lim_{\nu \nearrow \infty} \Delta(\alpha_I, \nu) = \infty$. As a result, there must exist ν' such that $\Delta(\alpha_I, \nu') = \Delta(\theta)$. Then, by Lemma 4, if $\Delta(\theta) \geq 0$ we must have $c(\alpha_I, \nu') > c(\theta)$. This implies that the most central investor has to be an expert.

However, as Figure 2 shows, not all investors with the highest screening ability are most central. Centrality is maximized at $\alpha = \alpha_I$ and an intermediate value of ν. Investors with high ν tend to have high reservation value, which implies they seldom engage in selling assets. Investors with low ν tend to have low reservation value, which implies they seldom engage in buying assets. The most central investor will have some intermediate valuation, ν^*, that will allow her to engage in trade both as a buyer and as a seller.

Not only does the most central investor have the highest screening ability, but a measure of investors above a certain centrality threshold do as well.

Proposition 3. *Consider a symmetric steady-state equilibrium $\{bid_n, ask_o, \Delta, \Phi_o, \Phi_n, s\}$. Then, $\alpha = \alpha_I$*

Figure 2: Trade centrality level curves



Notes: The figure presents trade centrality level curves –i.e. $c(\theta) = \tilde{c}$– as a function of asset valuation ν and screening ability α. The gray arrows show the direction by which the level curves increase. The arrows over the blue dotted lines show, for a given value of α, how centrality increases with ν. Finally, centrality is maximized at (α_I, ν^*).

for all investors type $\theta = (\alpha, \nu) \in \Theta$ such that $c(\theta) \geq \underline{c}$, where

$$\underline{c} := \frac{1}{2} \sup_{\theta \in \Theta} \{c(\theta)\} + \frac{1}{2} \sup_{\theta \in \Theta} \{c(\theta);\ \alpha \leq \alpha_{I-1}\}.$$

Proposition 3 implies that the group of investors that are most central in intermediating assets all possess the highest screening ability. In other words, those investors satisfying the condition $c(\theta) \geq \underline{c}$ endogenously form a core of a core-periphery market structure. They are the top-p investors in terms of centrality, where $p := \int \mathbb{1}_{\{c(\theta) \geq \underline{c}\}} dF$.

Finally, we note that investors belonging to the core of the trading network, as defined above, trade at a higher speed than investors in the periphery. The result follows as, in the model, speed is proportional to trade volume, as a result of asset holdings being restricted to $\{0, 1\}$.

5.2 Screening Ability and Middleman Activity

An important characteristic of asset intermediation is that an investor serves both sides of the market, as buyer and seller, acting as a middleman. In this section, we study how screening ability and centrality are related to an investor's role as a middleman.

Serving as a middleman, by buying and selling assets, and having a high centrality in terms of trade volume may seem to be closely related measures of intermediation—but they do not need to be. An investor can have a low volume of trade, and thus have low centrality, and still act as a middleman. Conversely, an investor can have a high volume of trade, and thus have high centrality, while predominantly engaging on one side of the market as buyer or seller. We show in this section that our environment generates that high centrality investors are not only experts, as we showed earlier, but are also middleman.

We characterize an investor's middleman activity by the extent the investor engages in buying versus selling. Let $s_n(\theta)$ denote the share of an investor's trades in which the investor is a buyer, what we refer to as their *buying share*. For θ satisfying $\Delta(\theta) < 0$, the buying share is not well defined because the investor does not trade, either as a buyer or a seller, so we set it to zero. For θ satisfying $\Delta(\theta) \geq 0$, the investor's buying share is

$$s_n(\theta) = \frac{\phi_n(\theta)\bar{q}_n(\theta)}{\phi_o(\theta)\bar{q}_o(\theta) + \phi_n(\theta)\bar{q}_n(\theta)} = \frac{[\mu + \lambda \bar{q}_o(\theta)]\bar{q}_n(\theta)}{[\eta + \lambda \bar{q}_n(\theta)]\bar{q}_o(\theta) + [\mu + \lambda \bar{q}_o(\theta)]\bar{q}_n(\theta)}, \tag{16}$$

where the second equality above follows by using the equilibrium conditions (12) and (13). An investor is more likely to serve as a buyer if she spends more time on average as a non-owner, given by ϕ_n, and conditional on being a non-owner, has a high ex-ante probability of trade, given by $\bar{q}_n$. In equilibrium, the buying share of an investor depends on the relative inflow and outflow of assets, as regulated by differences in η relative to μ.

We start by analyzing the connection between centrality and buying share in our model under complete information, where $\alpha_1 = \alpha_I = 1$. In this case, we are able to fully characterize the centrality-buying share relationship and provide a closed-form expression for the buying share of the most central investor. Then, we study numerically the case with private information, with results that conform to those obtained under complete information.

The next proposition relates an investor's centrality and buying share in the model with complete information.

Proposition 4. *Consider a symmetric steady-state equilibrium* $\{bid_n, ask_o, \Delta, \Phi_o, \Phi_n, s\}$ *in an economy with complete information—that is,* $\alpha_1 = \alpha_I = 1$*—and let* ν^* *be the most central investor. Given* ν_a *and* ν_b*, if either* $s_n(1, \nu_a) < s_n(1, \nu_b) \leq s_n(1, \nu^*)$ *or* $s_n(1, \nu_a) > s_n(1, \nu_b) \geq s_n(1, \nu^*)$ *then* $c(1, \nu_b) > c(1, \nu_a)$.

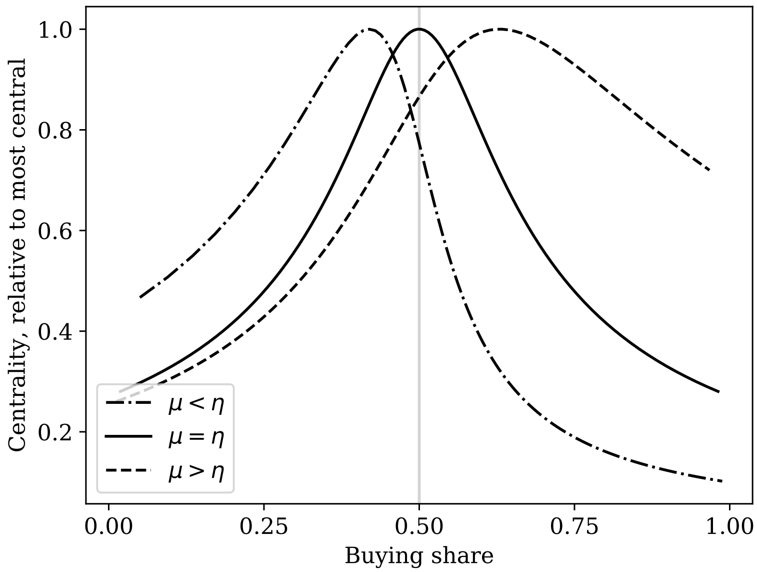
The proposition states that, under complete information, there is a unique mapping between an investor's centrality and buying share, both pinned down by the investor's asset valuation ν, and that as an investor's buying share moves away from that of the most central investor, the investor's centrality decreases. In other words, the proposition provides that, as a function of the investors' buying share, centrality is single-peaked, attaining its unique maximum at the buying share of the most central investor. Figure 3 provides numerical examples of this relationship, one where $\mu < \eta$, one where $\mu = \eta$, and one where $\mu > \eta$.

We can further characterize the buying share of the most central investor under complete information, as summarized in the next proposition.

Proposition 5. *Consider a symmetric steady-state equilibrium* $\{bid_n, ask_o, \Delta, \Phi_o, \Phi_n, s\}$ *in an economy with complete information—that is,* $\alpha_1 = \alpha_I = 1$*. Then, the most central investor is of type* $\theta^* = (1, \nu^*)$ *with* $\Delta(1, \nu^*) > 0$ *and buying share* $s_n(1, \nu^*) = \left[1 + \frac{\eta + \lambda \bar{q}_n(1,\nu^*)}{\mu + \lambda \bar{q}_n(1,\nu^*)}\right]^{-1}$.

The most central investor acts as a middleman, and the degree of middleman activity depends on the arrival rate of asset creation opportunities η relative to the depreciation rate μ. When $\eta = \mu$, we obtain that $s_n(1, \nu^*) = 1/2$. In this case, being a central investor entails a high degree of middleman activity, buying and selling assets in equal proportions. This is the case depicted as a solid line in Figure 3.

Figure 3: Buying share and centrality, under complete information



Notes: We set $\alpha = 1$ for all investors and $r = 0.05$, $\eta = 1/8$, $\mu \in \{\eta/5, \eta, 5\eta\}$ $\lambda = 4.0$, $\xi_0 = 0.5$, and $\nu \sim U[.5, 3.5]$. For each case, we normalize the centrality of any investor by dividing by the centrality of the most central investor.

When $\eta > \mu$ the buying share of the most central investor is lower than $1/2$. There are direct and indirect effects of increasing η on the buying share of an investor. The direct effect is that a higher η provides more frequent opportunities for an investor to obtain an asset without the need of buying it. This effect acts to reduce the buying share of an investor. The indirect, general equilibrium effect is that, as a result of a higher η, the market will have a higher population of owners. In turn, this implies that it is now easier for an investor to buy an asset in the market, thus forcing the buying share of the investor to increase. Although these two effects push the buying share in opposite directions, we can show that when $\eta > \mu$ the direct effect dominates and an investor is central in trade when they sell often. This forces the buying share of the most central investor to be below $1/2$ (dash-dotted line in the Figure 3). In particular, when η diverges to infinity, $s_n(1, \nu^*) \to 0$. Here, as asset creation opportunities arrive instantaneously, the most central investor never buys assets.

When $\eta < \mu$ the same intuition holds but in the opposite direction, forcing the buying share of the most central investor to be above $1/2$ (dashed line in the Figure 3). In particular, when μ diverges to infinity, $s_n(1, \nu^*) \to 1$. Here, as assets depreciate instantaneously, the most central investor's buying share equals one.

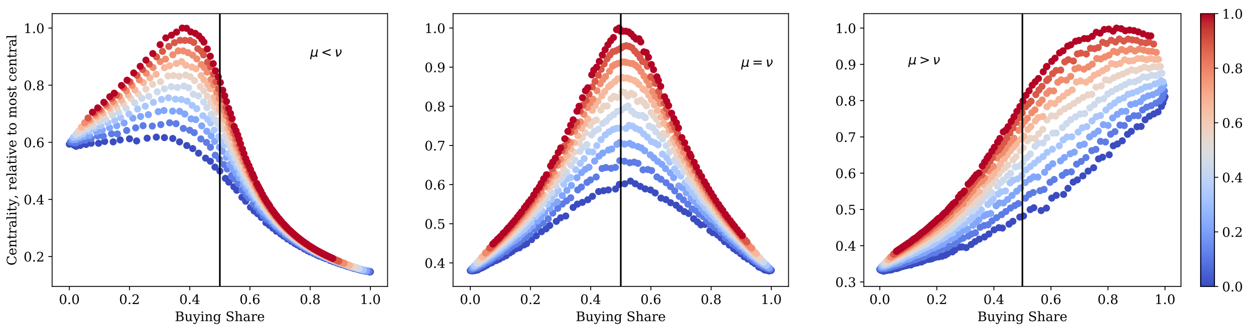
Private information and information asymmetries complicate things considerably and prevent us from providing results as tight as those obtained under complete information. The next proposition provides an important partial result for the connection between an investor's centrality and buying share.

Proposition 6. *Consider a symmetric steady-state equilibrium* $\{bid_n, ask_o, \Delta, \Phi_o, \Phi_n, s\}$, *and two investor types,* $\theta_a = (\alpha_a, \nu_a)$ *and* $\theta_b = (\alpha_b, \nu_b)$, *satisfying* $\alpha_a > \alpha_b$. *If the investors have the same share buying,* $s_n(\theta_a) = s_n(\theta_b) \in (0, 1)$, *the investor with higher screening ability is more central,* $c(\theta_a) > c(\theta_b)$.

Unlike in the complete information case, now an investor's centrality and buying share depend on her asset valuation ν and screening ability α. As a result, it is possible that investors with different centrality have the same buying share. Among those with same buying share, Proposition 6 provides that centrality is higher for the investor with an information advantage.

We show by way of numerical examples that the main results obtained under complete information, presented in Proposition 4, apply more generally. Figure 4 illustrates, for different $\{\eta, \mu\}$ configurations, that centrality is single peaked in the space of buying shares and as the buying share moves away from that of the most central investor, centrality falls. Notice also how the plot confirms the result in Proposition 6: for a given level of buying share, an investor's centrality increases with her screening ability α.

Figure 4: Buying share and centrality, under heterogeneous screening ability



Notes: We set $\alpha \sim U[0,1]$, $r = 0.05$, $\eta = 1/8$, $\mu \in \{\eta/5, \eta, 5\eta\}$ $\lambda = 4.0$, $\xi_0 = 0.5$, and $\nu \sim U[.5, 3.5]$. For each case, we normalize the centrality of any investor by dividing by the centrality of the most central.

5.3 Screening Ability and Trading Network

Several papers studying financial networks, such as Green et al. (2007), Bech and Atalay (2010) and Hollifield et al. (2017), document that central traders are not only involved in a large share of trade volume, as measured by (15), but are also more connected to other investors. A natural question then, in the context of our theory, is if investors with high screening ability also have a stronger connection with counterparties.

We use two measures of connectedness: *degree* and *strength*. The degree is simply the total number of unique counterparties an investor has, which we define as

$$np(\theta) = np_{out}(\theta) + np_{in}(\theta) = \int \mathbb{1}_{\{q(\theta,\hat{\theta})>0\}} dF(\hat{\theta}) + \int \mathbb{1}_{\{q(\hat{\theta},\theta)>0\}} dF(\hat{\theta}), \tag{17}$$

for an investor of type θ. The out-degree of an investor type θ, denoted by $np_{out}(\theta)$, is the number of unique counterparties that buy from the investor; and the in-degree, denoted by $np_{in}(\theta)$, is the number of unique counterparties that sell to the investor. The degree of the investor is the sum of the in- and out-degrees.

The following lemma shows that non experts—defined as investors with $\alpha = 0$—trade with a smaller set of counterparties than those investors with some screening expertise, and therefore have a lower degree in the trade network.

Lemma 5. *Consider a symmetric steady-state equilibrium $\{bid_n, ask_o, \Delta, \Phi_o, \Phi_n, s\}$ and some investor type $\theta \in \Theta$ such that $\Delta(\theta) > 0$. We have that $np(\theta) = 1$ if $\alpha = \alpha_I > 0$, and $np(\theta) < 1$ if $\alpha = 0$. Moreover, if the types $\theta = (\alpha, \nu)$ and $\hat{\theta} = (\hat{\alpha}, \hat{\nu})$ satisfy $\Delta(\theta) = \Delta(\hat{\theta}) > 0$ and $\alpha = 1 > \hat{\alpha} = 0$, then $np_{out}(\theta) > np_{out}(\hat{\theta})$ and $np_{in}(\theta) > np_{in}(\hat{\theta})$.*

Investors with a negative reservation value do not hold or trade assets in equilibrium. Among the investors with positive reservation value, an investor trades with every other investor if they are a screening expert.[14] On the other hand, non-experts distort trade due to private information and do not trade with a set of other investors, $np(\theta) < 1$.

The following Proposition follows from Lemma 5 to show that the average degree of connectedness for periphery investors is lower than for core investors.

Proposition 7. *Given a symmetric steady-state equilibrium $\{bid_n, ask_o, \Delta, \Phi_o, \Phi_n, s\}$, we have that*

$$\frac{\int_{\theta \notin \bar{\Theta}} np(\theta) f(\theta) d\theta}{\int_{\theta \notin \bar{\Theta}} f(\theta) d\theta} < \frac{\int_{\theta \in \bar{\Theta}} np(\theta) f(\theta) d\theta}{\int_{\theta \in \bar{\Theta}} f(\theta) d\theta} = 1,$$

where $\bar{\Theta} := \{\theta \in \Theta;\ c(\theta) \geq \bar{c}\}$ and $\bar{c}$ is defined as in Proposition 3.

Investors at the core of the trade network trade with more unique counterparties than those in the periphery of the trade network.

A shortcoming of the degree measure is that it considers only binary connections. That is, the measure counts equally all counterparties, not considering the frequency of trade with them. An alternative is to use strength as measure of connectedness. Instead of simply adding up the number of unique counterparties of the investor, as we do with degree, the strength measure weights the connection to each counterparty. This approach is consistent with the method followed in Physics to study complex networks, as in Barrat, Barthelemy, Pastor-Satorras, and Vespignani

[14] In fact, all investors with $\alpha > 0$ trade with all other investors in the stationary equilibrium since almost surely they will be informed with every other trader they meet both on the buy and sell side.

(2004). In our case, a natural weight to use is the probability of trade between two investors,

$$\begin{aligned} st(\theta) &= \frac{\phi_o(\theta)}{f(\theta)} st_{out}(\theta) + \frac{\phi_n(\theta)}{f(\theta)} st_{in}(\theta) \\ &= \frac{\phi_o(\theta)}{f(\theta)} \int \frac{\phi_n(\hat{\theta})}{f(\hat{\theta})} q(\theta,\hat{\theta}) dF(\hat{\theta}) + \frac{\phi_n(\theta)}{f(\theta)} \int \frac{\phi_o(\hat{\theta})}{f(\hat{\theta})} q(\hat{\theta},\theta) dF(\hat{\theta}). \end{aligned} \tag{18}$$

The out-strength of an investor type θ, denoted by $st_{out}(\theta)$, is the integral of unique counterparties that buys from the investor weighted by the probability of the trade, which is the probability the counterparty is a non-owner, $\phi_n(\hat{\theta})/f(\hat{\theta})$, times the probability of trade $q(\theta, \hat{\theta})$. Similar intuition holds for the in-strength. The strength of an investor type θ is the weighted sum of the in- and out-strength, where the weights are the probability the investor is a seller, $\phi_o(\theta)/f(\theta)$, and the probability she is a buyer, $\phi_n(\theta)/f(\theta)$.

Since the strength of an investor's connections are weighed by trade probabilities, it has very close relationship with centrality. In particular, the following proposition holds.

Proposition 8. *Consider a symmetric steady-state equilibrium $\{bid_n, ask_o, \Delta, \phi_o, \phi_n, s\}$ and some investor type $\theta \in \Theta$ such that $\Delta(\theta) > 0$. Then, $st(\theta) = \frac{2Vol}{\lambda} c(\theta)$.*

An investor's network strength is interchangeable with their centrality up to a positive constant. In particular, using Proposition 2, we have that the most connected investor according to network strength must be an expert with $\alpha = \alpha_I$. Hence, our measure of centrality lines up well with network-based concepts of centrality.

5.4 Screening Ability and Rents

We close Section 5 by studying the relationship between screening ability and rent extraction. The theoretical literature has shown that intermediation activity in OTC markets can be linked to investors with a better ability to extract rents. Our theory also delivers a related prediction, but only when considering the lifetime discounted value of being able to extract higher rates. That is, our theory provides that the value functions $V_n(\theta)$ and $V_o(\theta)$ are increasing in screening ability α. The next proposition establishes the main result of this section.

Proposition 9. *Consider a symmetric steady-state equilibrium $\{bid_n, ask_o, \Delta, \Phi_o, \Phi_n, s\}$, an investor type $\theta = (\alpha, \nu)$ and $\hat{\theta} = (\hat{\alpha}, \nu)$ such that $\Delta(\theta) > 0$ and $\hat{\alpha} > \alpha$. Then, the value functions V_o and V_n, defined in equations (5) and (6), satisfy $V_o(\hat{\theta}) > V_o(\theta)$ and $V_n(\hat{\theta}) > V_n(\theta)$.*

The results in the proposition follow as an investor with the higher α can always replicate the choices of an investor with lower α. As a result, the value function of the investor with the higher α lies above the value function of the investor with the lower α. As intuition suggests, the proposition provides that the discounted value of rents of experts is higher than the discounted value of rents of non-experts. Given our previous results that experts populate the core of the trading network, we can conclude that investors at the core extract the largest discounted rents from intermediation. A related result is presented in the next lemma.

Lemma 6. *Consider a symmetric steady-state equilibrium* $\{bid_n, ask_o, \Delta, \Phi_o, \Phi_n, s\}$*, and let the types* $\theta = (\alpha, \nu)$ *and* $\hat{\theta} = (\hat{\alpha}, \hat{\nu})$ *satisfy* $\Delta(\theta) = \Delta(\hat{\theta})$ *and* $\alpha > \hat{\alpha}$*. Then,* $\pi_o(\theta) > \pi_o(\hat{\theta})$ *and (ii)* $\pi_n(\theta) \geq \pi_n(\hat{\theta})$*, with strict inequality if* $\Delta(\theta) = \Delta(\hat{\theta}) > 0$*.*

The lemma provides that, for a given valuation $\Delta(\theta) = \Delta(\hat{\theta})$, the expected instantaneous trading profits are increasing in expertise α. In other words, $\pi_n(\theta)$ and $\pi_o(\theta)$ are increasing in α in partial equilibrium.

Interestingly, the result in Lemma 6 does not necessarily extend to general equilibrium; this occurs as we cannot sign the relationship between the reservation value $\Delta(\theta)$ and α. As discussed in Section 5.1, $\Delta(\theta)$ can be increasing or decreasing in α, depending on whether the investor benefits from screening expertise by either trading frequently or at high prices. The fact that we cannot sign the dependence of $\Delta(\theta)$ on α, implies that we cannot sign any other instantaneous rent object. That is, on top of instantaneous profits $\pi_n(\theta)$ and $\pi_o(\theta)$, the model does not have robust predictions about prices, markups, and bid-ask spreads, among others.

6 Empirical Analysis

In this section, we empirically evaluate the model's predictions regarding the relevance of private information in shaping trade outcomes and market structure using data on the OTC market for Credit Default Swaps (CDS). The CDS market is a useful laboratory since, as we discuss in this section, it features a distinct core-periphery structure. The key mechanism in the model that drives how an investor's screening ability determines their role in intermediation is its impact on their probability of trade. Leveraging regulatory reporting requirements, we first show that information disclosure increases an institution's probability of trade with the market's periphery but has little to no effect on trade with the core. These results are consistent with our model's predictions that the core is comprised of experts while the periphery less so. We then examine how these effects on the probability of trade shape intermediation in the market as a whole.

6.1 The CDS data

The market for CDS is large and active, with a daily notional volume around $2 trillion. Generally a CDS contract, called a single-name CDS, involves an agreement in which the buyer of protection makes regular payments to the seller of protection in exchange for a contingent payment from the seller upon a credit event (e.g. nonpayment of debt) on a specified reference entity (e.g. a single corporate bond issue). In our analysis, we focus on trades of US CDS indexes, which are bundles of US single-name CDS.

We use trade-level CDS data from the Trade Information Warehouse (TIW) of the Depository Trust and Clearing House Corporation (DTCC). Our sample includes trades from the 1st quarter of 2013 to the 4th quarter of 2017, a total of 19 quarters or 246 weeks. The Dodd-Frank Wall Street Reform and Consumer Protection Act requires real-time reporting of all swap contracts to a registered swap data repository (SDR), such as the DTCC, and makes the reported data available to appropriate prudential regulators.[15] For each transaction, we observe the day of the trade, the name of the buyer and seller, the reference entity (or series of the index), and other details of the contract (e.g. notional amount, initial payment, etc.).

We focus on CDS indexes for several reasons: CDS indexes are standardized, unlike single-name CDS; they are centrally cleared so that counterparty risk does not determine counterparty choice, as discussed in Du, Gadgil, Gordy, and Vega (2017); and they have a higher frequency of trade than single-name CDS, while still trading OTC. High trade frequency is particularly important for us because it allows us to control for unobservable index, institution, and time period characteristics using fixed effects, whenever it is needed. Finally, our main results focus on the US because the data only covers trades that either have a party or reference entity regulated domestically. Since US indexes are mostly traded by US firms or subsidiaries of foreign firms, they are more likely to be regulated by the Federal Reserve and be included in our data.

A total of 4,124 institutions trade US CDS indexes in our sample period, accounting for 369,527 transactions. We observe trades of 36 US CDS-index classes. The trade activity is concentrated in North American CDS indexes (CDX.NA) and Commercial Mortgage Backed Security Indexes (CMBX). These two classes account for 89% of all US trades in our data. The average number of trades in a given week for any of the indexes across all institutions is 1,502. Even though CDS indexes are more liquid than single-name CDS, they are still traded relatively infrequently. As a result, our regressions will use observations of trade at a weekly frequency.

The market for CDS indexes is concentrated. To see this, for each institution we calculate the percentage of all trades in which they participate either as a buyer or seller—consistent with the centrality measure used in Section 5—then rank them from largest to smallest. We find that the top-five institutions participate in 92% of trades. There is also a noticeable discontinuity in centrality from the top-5th to the top-6th institution. The institution ranked 5th has a centrality measure 102% higher than the institution ranked 6th, while the institution ranked 4th has a centrality measure only 19% higher than the institution ranked 5th. Based on these findings, we associate the top-five institutions in the data with the set of core institutions in our theory.

[15]See Sections 727 and 728 of The Dodd-Frank Wall Street Reform and Consumer Protection Act. As a prudential regulator, members of The Federal Reserve System have access to the transactions and positions involving individual parties, counterparties, or reference entities that are regulated by the Federal Reserve. The raw CDS trade data is stored at the Federal Reserve Board (FRB) servers and are downloaded from the DTCC regulatory portal.

6.2 The role of information in shaping up trade outcomes

Our empirical strategy is the following. A subgroup of investors, which includes managers from banks, insurance companies, broker-dealers, pension funds, and corporations, are required to file the 13-F form to the Securities and Exchange Commission (SEC). The form contains the end-of-quarter holdings of all securities regulated by the SEC, which mostly consist of equities that trade on an exchange and equity options, but also shares of closed-end investment companies and certain debts. The SEC makes the 13-F filing public immediately after it is filed, and so market participants know detailed portfolio information about 13-F filers. We then study how a 13-F filing impacts an investor's probability of trade with core versus periphery counterparties in the OTC market for CDS indexes.

CDS are a primary way for institutions to hedge against risk in their portfolio, so implicit in our empirical strategy is the interpretation that a 13-F filing (at least imperfectly) reveals information about an investor's trading needs in the CDS market. This interpretation has been supported in the literature both theoretically and empirically. For instance, the seminal model of Merton (1974) illustrates that a firm's credit spreads and equity prices are fundamentally linked through the firm's optimal choice of capital structure. If credit and equity assets have correlated returns, then the demand for an asset written on a firm's debt, such as a CDS, is correlated with the holdings of their equity—which the 13-F reveals. Empirically, papers such as Campbell and Taksler (2003), Blanco, Brennan, and Marsh (2005), Lonstaff, Mithal, and Neis (2005), Zhang, Zhou, and Zhu (2009) and Forte and Pena (2009), have found a correlation between debt and equity returns.

6.2.1 Model predictions about the effects of a 13-F filing

Before moving to the empirical test, we develop a set of model predictions about the effects of information disclosure by 13-F filings, summarized in Proposition 10. We consider a version of the baseline model above, but assume that a fraction of investors, which we label as "13-F investors", have to publicly disclose information at random future dates through a 13-F filing. The distribution of types across 13-F and non-13-F investors is given by F_{13F} and F_{n13F}, with both satisfying the same assumptions we imposed on the distribution F. We use $\omega_{13F} \in [0,1]$ to denote the fraction of 13-F investors, so that $F = \omega_{13F} F_{13F} + (1 - \omega_{13F}) F_{n13F}$. It is common knowledge who the 13-F investors are but not their utility type.

Starting from some known future date t_0, each 13-F investor draws a filing window $[T_0, T_0 + T]$, with $T_0 \geq t_0$ and $T > 0$, where T_0 is drawn from a Poisson distribution with parameter $\gamma > 0$. Investors filing dates are independent and become known in t_0. At time T_0, the investor chooses the filing date $\tilde{T}_0 = T_0 + \tilde{t}$, where $\tilde{t} \in [0, T]$ denotes the filing delay. We allow investors to delay the 13-F filings to be consistent with the 45-days delay window given by the SEC, described more in Section 6.2.2.

The information content in a 13-F filing is imperfect. In any meeting after an investor's filing, the 13-F report perfectly reveals the filer's utility type to the counterparty with probability $\rho \in (0, 1]$. With probability $1 - \rho$, the 13-F report is uninformative. This shock is independent and identically distributed across 13-F investors and meetings, and independent from other shocks.

An equilibrium is defined in a similar fashion to that provided in 1. The key difference is that the objects are time dependent. If we were to consider a steady-state equilibrium, then all 13-F would have been filed and we would not be able to study the impact of filing on trade probabilities. Because we do not consider a steady state equilibrium, the measure of 13-F investors who have filed changes over time making all the other equilibrium objects time dependent as well.

We are interested in studying how a filing impacts a 13-F investor's probability of trade with a "core" investor relative to a "periphery" investor. We define the set of core investors as those with a centrality measure in the top-p percentile as defined in Proposition 3. The following proposition establishes our main set of testable predictions regarding the effects of information disclosure on trade probability.

Proposition 10. *All 13-F investors are weakly better off delaying the filing (choosing $\tilde{t} = T$), and the 13-F investors with strictly positive probability of trade are strictly better off delaying the filing date. Moreover, when a 13-F investor with strictly positive reservation value files the 13-F, there is*

(i) a strict increase in her probability of trade with periphery investors,

(ii) no change in her probability of trade with core investors, and, as a result,

(iii) a higher increase in trade probability with periphery investors than with core investors.

The first result in Proposition 10 is that information revelation hurts a 13-F investor when she trades. Thus, if a 13-F investor expects to trade in the filing window, she delays filing up to the maximum allowed. In the data, most institutions delay their filing up to 45 days allowed.

The second result is divided into items (i)-(iii). Items (i) and (ii) establish that information disclosure weakly increases an investor's probability of trade. Intuitively, more information reduces distortions that follow from information asymmetries and destroy trades. Further, and more importantly, part (iii) establishes that *information disclosure disproportionately increases an investor's probability of trade with the periphery relative to the core*. This result stems from the key prediction of our model. Since the theory predicts that those investors who endogenously populate the core have an informational advantage, public information disclosure will impact them less. The result does not rely on the utility type or screening ability of the 13-F investor, or in turn their centrality.[16] We test these predictions in the remainder of the section.

[16]The predictions of Proposition 10 are also likely robust to adding additional heterogeneity across investors, for instance in their contact rates, λ. However such a model would predict differences in the probability of trade across investors with the same screening ability but different contact rates. In this case, the results would need to be re-stated by normalizing by the unconditional probability of trade with core and periphery investors.

6.2.2 The 13-F data

Congress passed Section 13(f) of the Securities Exchange Act in 1975 in order to increase the public availability of information regarding the security holdings of institutional investors. Under Section 13(f), any registered investment manager with discretion over its own or a client account with an aggregate fair market value of more than $100 million in Section 13(f) securities must file a 13-F form. The 13-F form lists the holdings of Section 13(f) securities, which primarily includes US exchange-traded stocks (such as those traded on the NYSE and NASDAQ), stock options, shares of closed-end investment companies, and shares of exchange-traded funds (ETFs).[17] Importantly, CDS are not included in the list of 13(f) securities.

The SEC makes 13-F filings publicly available through its Electronic Data Gathering, Analysis, and Retrieval (EDGAR) program. The identity of the institutions that must file a 13-F form is known by market participants. This implies that when a report is filed, counterparties of 13-F filers possess detailed portfolio information when trading. Since CDS indexes are a way for institutions to hedge against risk in their portfolio, we interpret the information provided to the market in the 13-F form as information related to the trading needs of investors on CDS.

When filing a 13-F form, institutions are required to list their portfolio ownership of all Section 13(f) securities as of the last trading day of each quarter, which we label the report date. However, institutions have the discretion to delay reporting and file a 13-F form up to 45 days past the report date. We label the day in which institutions actually file the filing date. The 45-day delay rule is designed to protect investors from copycatting and front-running. From EDGAR, we observe the filing date and the unique Central Index Key (CIK) of the filing manager, which gives us the institution name and is used to link filing institutions to those trading CDS.

6.2.3 The merged dataset

Table 1 provides summary statistics of the merged dataset. Details on merging are available in Appendix B.1. We consider three different samples. Our preferred sample includes only institutions that filed at least once in the sample period, which we label as *filers*, and trades of US CDS indexes. Since in our regressions we control for fixed effects at the institution level, narrowing the sample to include only filers is enough to identify the effect of a 13-F report in the time period around a filing. We provide more details on identification in the next section. We also report results for the sample that includes all traders, regardless of filing status, and indexes.

Of a total of 4,124 institutions trading US CDS indexes in our sample period, 52 filed a 13-F in at least one quarter. Most institutions are not filing 13-F reports. Additionally, not all filing institutions filed in every quarter in our sample; out of the 52 institutions that filed in at least one

[17] See https://www.sec.gov/fast-answers/answers-form13fhtm.html for a complete list of 13(f) securities and other institutional details.

Table 1: CDS trades and 13-F filings, summary statistics

	Sample	
	US indexes, filers	US indexes, all institutions
Number of institutions	52	4,124
% that file in every quarter	51.9	0.7
Number of trades	133,901	369,529
% involving two *filers*	4.6	1.7
% in which at least one institution filed in prev. week	2.2	0.8
% in which at least one institution filed in previous 2 weeks	4.3	1.6
average trades per week	463.0	1,502.1
average trades per week, per institution	8.9	0.4
Number of index-classes traded	36	36

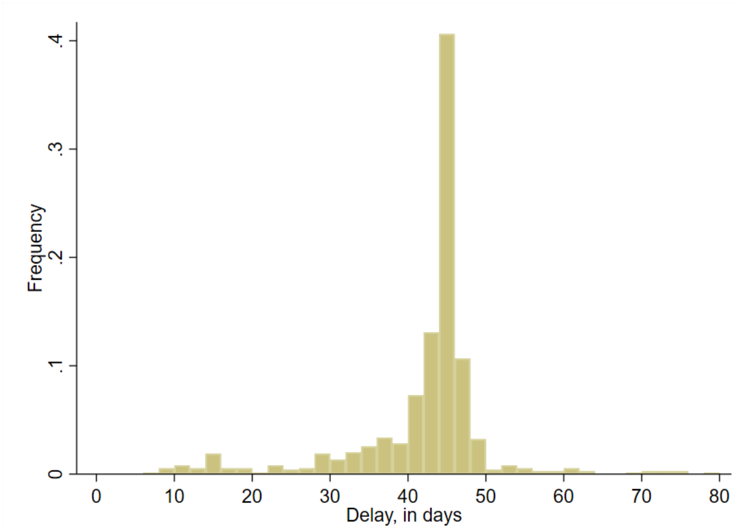
quarter, a little over half filed in every quarter. To capture any endogeneity that concerns selection into filing a 13-F, we will show results that control for fixed effects at the institution-quarter level. That is, we will only exploit variation within a quarter around filing dates.

Filers make up a small fraction of institutions trading CDS indexes, but they participate in a large fraction of trades. Of the 369,527 CDS-index transactions we observe, 36.24% involve at least one *filer*. This is in line with the idea that 13-F filers tend to be larger institutions, and the amount of trade activity provides us with enough observations for identification. Although large, almost all filing institutions are not among the most central in terms of trade volume for the overall sample of institutions trading CDS. In fact, only a small fraction of trades of filers include a filer among the top-5 investors in terms of trade volume. The model predictions regarding 13-F filing hold both for periphery and core investors, as stated in Proposition 10, but the low fraction of trades by filing investors that are also central provide reassurance regarding the lack of relevance of potential mechanisms linked to the market volume of the filing institution.

Typically, only one of the two participants in a transaction is a filer; only 4.6% involve a trade between two filers in the main sample. Among the trades involving at least one filer, the share in which buyer or seller filed is split roughly even. Table 1 also shows the extent of trades we observe in the time period recently following a 13-F report. Since our theoretical predictions are all local in nature, our tests focus on trading activity during this time period. Of the total trades we observe, 0.81% involve an institution that filed a recent 13-F report, that is within the previous week, and 1.56% involve an institution that filed within the previous two weeks.

As mentioned above, institutions have the option to delay filing the 13-F form for up to 45 days past the report date. We define a delay as the difference in days between the report and filing date. Figure 5 presents the distribution of delays across investors. All institutions delay their filings, with a minimum delay of 6 days. Also, 18% of filings have a delay over the 45-day limit. This can occur for two reasons. If day 45 falls on a weekend or holiday the deadline is extended until the following business day. Institutions can also apply to extend the standard delay period; however, delaying over 48 days is rare and accounts for only 5.4% of filings.

Figure 5: The distribution of delays



Notes: Sample includes all filings by institutions trading US credit default swap indexes by regulated institutions or those trading CDS indexes on regulated institutions in the sample period, 2013Q1-2017Q4. The figure presents the histogram of filing delays, defined as the difference between the actual and official filing dates.

Figure 5 reveals that most filings occur around the 45-day limit: 66% of delays are in a window of 42 to 48 days. Delaying filing until the end of the report period is consistent with our Proposition 10 which states that investors strategically delay their filings. Notwithstanding this, our model cannot explain variation in delays. Although institutions file form 13-F for motives exogenous to CDS trading, it is possible that delaying a filing is correlated with the institution's CDS trading activity. Section 6.2.6 discusses how alternative theories relate to our empirical results and, in particular, discusses theories with endogenous delays.

6.2.4 The effects of a 13-F filing on CDS trade

In this section, we test the predictions described in Proposition 10 that a 13-F filing (i) strictly increases the probability of trading with periphery institutions, (ii) weakly increases the probability of trade with core institutions, and (iii) shifts trade towards periphery institutions. To do so, we estimate the following linear model:

$$Y_{ijt} = \beta \frac{F_{j,t-1}}{Frequency} + \text{Fixed Effects}_{jit} + \epsilon_{jit} \tag{19}$$

where i denotes a CDS-index class (e.g. CDX.NA.IG), j denotes an institution, and t denotes a time period, which we set to be equal to a week. The variable Y_{ijt} represents the outcome of interest which is one of two possible dummy variables, D^p_{ijt}, D^c_{ijt}, where p stands for periphery and c stands for core. The variable D^p_{ijt} is a dummy for a trade by institution j involving CDS index i in week t trading with any periphery institution. Similarly D^{core}_{ijt} is a dummy for trade by institution j involving CDS index i in week t trading with any core institution.

The coefficient of interest is β—the coefficient on the dummy variable $F_{j,t-1}$, which equal to one if institution j filed a 13-F report in the week previous to t. As we discuss in subsection 6.2.1, we normalize the dummy $F_{j,t-1}$ in each regression by the frequency of trade in the sample (i.e. frequency of trade involving core and frequency of trade involving periphery investors). This normalization allows us to control for other determinants of trade centrality, such as trade speed as discussed in Üslü (2019) and Farboodi, Jarosch, and Shimer (2017), that would differentially affect the probability of trade with core relative to periphery investors. Under the normalization, we can compare the coefficient β across specifications.

Identification comes primarily from (i) comparing trade activity by the same institution in weeks following a report versus not, and (ii) comparing trades within a week across institutions that filed in the week before versus those that did not. For (i) we have variation in the weeks and indexes that institutions trade relative to the weeks where they file. For (ii) we have variation in the weeks where different institutions file. The large transactional data allow us to control for many unobservable correlations using a combination of fixed effects on institutions, weeks and indexes. One potential concern is endogeneity bias that stems from the filing requirements of Form 13-F causing selection on unobservables that are correlated with trading activity in CDS markets. This type of bias is unlikely to effect our results since the number of institutions required to file Form 13-F is considerably larger than those that file *and* trade CDS indices.[18] However, we address this concern by narrowing our baseline sample to only filers.

Table 2 reports our baseline results. Column (1) includes fixed effects for week-index pairs and institutions, and restricts the sample to filers and US indexes. We find that the probability of trade with periphery institutions increases by 22.5% in the week after a 13-F filing. We also find that the probability of trade with core institutions increases, by 9.6%. Both effects are statistically significant and positive, consistent with parts (i) and (ii) in Proposition 10; a 13-F report leads to increased trading activity in the week following the report. However, part (iii) of 10 suggests that the information revelation should shift the probability of trade towards periphery institutions. The third panel of Table 2 shows the results of testing the difference in the first two coefficients. We find that the difference is positive and significant; a 13-F report increases trade with periphery institutions by 12.9% relative to core institutions.

Columns (2) and (3) in Table 2 test for the robustness of our results to the sample selection. In column (2) we extend our baseline sample to include non-US indexes, and in column (3) we

[18]For instance, there are 5,560 financial institutions that filed at least one Form 13-F between 2013 and 2017, while only 52 institutions in our dataset trade CDS indices and filed form 13-F at least once in the same sample period.

Table 2: Impact of a 13-F filing on trade

	US index, filers	All index, filers	US index, all inst.	US index, filers
	(1)	(2)	(3)	(4)
Trade with Periphery, β^p	0.225***	0.125***	0.501**	0.137*
	(0.075)	(0.046)	(0.198)	(0.074)
R-squared	0.176	0.167	0.107	0.198
Trade with Core, β^c	0.096*	0.095**	-0.002	-0.010
	(0.054)	(0.038)	(0.128)	(0.053)
R-squared	0.182	0.177	0.069	0.204
Test on difference, $\beta^p - \beta^c$	0.129*	0.030	0.498**	0.147**
	(0.073)	(0.042)	(0.209)	(0.072)
Fixed Effects				
Week − index	yes	yes	yes	yes
Institution	yes	yes	yes	no
Institution − quarter	no	no	no	yes
Observations	460,512	1,100,358	36,522,144	460,512

Notes: Sample includes trades of US credit default swap indexes by regulated institutions or those trading CDS indexes on regulated institutions, that filed a 13-F report at least once in the sample period, 2013Q1-2017Q4. The independent variable is a normalized dummy, where the dummy is equal to one if institution j filed a 13-F in the previous week. The two dependent variables are dummies if institution j traded CDS index i in week t with a periphery and core institution, respectively. Test on difference: tests whether the difference in the coefficients is equal to zero. Standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

extend our sample to also include all institutions, regardless of filing status. The results remain consistent with those presented in column (1). Specifically, extending the sample to include all institutions implies the effects of a 13-F on trade with core institutions disappears and leads to a nearly 50% increase in the probability of trade with periphery institutions.

In column (4), we report results that control for fixed effects at the institution-quarter level. In our sample of filers not all institutions filed in every quarter. This may be resulting from either an error in our process of matching 13-F reports to CDS trades or in variation in the size the institution's 13(f) portfolio from quarter to quarter that could, in principle, bias in our results.[19] Adding institution-quarter fixed effects address both of these concerns by limiting our identifying variation to weeks within an institution's filing quarter. Doing so leads to results that are also in line with columns (1)-(3). A 13-F report increases the probability of trade with periphery institutions relative to core institutions by 14.8%.

In Table 3, we examine the effect of a 13-F report on trade in the following one-, two-, three- and four-week windows. While Proposition 10 only concerns the effects of a 13-F in the time period immediately following a report, we are interested in investigating the persistence of the shock. Column (1) repeats the results from column (4) in Table 2, which controls for fixed effects by week-index and institution-quarter. The positive effect of 13-F filing on the probability of

[19]For instance, we may be unable to recover the filing date if the institution manager changes since the last filing – as filing is manager specific– or if she files the report jointly with another manager, which can occur if both managers belong to a bigger corporation.

Table 3: Impact of 13-F filing on trade, varying lag lengths

	1 week	2 weeks	3 weeks	4 weeks
	$F_{j,t-1}$	$F_{j,t-2}$	$F_{j,t-3}$	$F_{j,t-4}$
	(1)	(2)	(3)	(4)
Trade with Periphery, β^p	0.137*	0.146**	0.029	-0.058
	(0.074)	(0.058)	(0.052)	(0.049)
Trade with Core, β^c	-0.009	-0.014	-0.006	-0.037
	(0.053)	(0.042)	(0.038)	(0.035)
Test on difference, $\beta^p - \beta^c$	0.147**	0.160***	0.035	-0.021
	(0.072)	(0.057)	(0.051)	(0.048)
Fixed Effects				
Week − index	yes	yes	yes	yes
Institution − quarter	yes	yes	yes	yes
Observations	460,512	458,640	456,768	454,896

Notes: Sample includes trades of US credit default swap indexes by regulated institutions or those trading CDS indexes on regulated institutions, that filed a 13-F report at least once in the sample period, 2013Q1-2017Q4. The independent variables, (1) $F_{j,t-1}/Frequency$, (2) $F_{j,t-2}/Frequency$, (3) $F_{j,t-3}/Frequency$, and (3) $F_{j,t-4}/Frequency$ are normalized dummies, where the dummies are equal to one if institution j filed a 13-F within the previous week, two weeks, three weeks, and four weeks respectively. The two dependent variables are dummies if institution j traded CDS index i in week t with a non-top-5 and top-5 institution, respectively. Test on difference: tests whether the difference in the coefficients is equal to zero. Standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

trade with periphery institutions holds up to two weeks after the filing, but it is not present in the three- and four-week windows. The effect of a 13-F on the probability of trading with a core institutions remains insignificant and with a point estimate close to zero. The difference between the two, or the effect of a 13-F on the probability of trade with periphery versus core institutions is significant and positive up to two weeks after the filing date, with a difference in the change of trading probability of nearly 15%, but vanishes after two weeks. These results are consistent with our theory. As we increase the window length, we also increase the number of trades that we consider as related to filing a report. In theory, by doing so we are adding trades that are less correlated with the revelation of information, thus adding noise and eventually breaking the link between trade activity and filing.

While we find a significant impact of a 13-F filing on trade with periphery institutions, we find no effect on trade with core institutions. The coefficient estimates for trade in the one or two weeks after a 13-F filing versus before are statistically the same (as shown in the middle panel of Table 7). We test the difference in the probability of trade and find a significant, positive impact of a 13-F on trade with periphery institutions relative to core institutions, which verifies the estimates effects from Tables 2 and 3.

Consistent with Proposition 10, most institutions filing form 13-F delay their filing as much a possible, filing close to the 45-day limit (66% of filings have a delay between 42 and 48 days). While our model can account for most delays, it cannot account for delays of shorter duration, as the model implies that institutions have strong incentives to delay filings up to the maximum allowed. For filings with short delays, this raises the question whether the timing of a filing is correlated with particular trades in CDS that the institution intends to do around the filing date. One way to address these concerns is to study the effect of filing on trade activity around the maximum filing delay. Since the 45-day constraint applies to all institutions filing form 13-F in any quarter and since the 45th day of any quarter is arguably uncorrelated with any particular investors' trading motives in CDS, we can use the delay limit as exogenous variation in filing. Table 6 in Appendix B.2 repeats our baseline regression, but restricts to trades with filing delays of 42 to 48 days.[20] As the table shows, we find that our results strengthen in this case, providing reassurance of the relevance of the information mechanism in determining trade outcomes.

Another concern is that institutions simply trade more around filing events and that the dummy for trade after filing is capturing this effect. In Table 7 in Appendix B.3 we study this, by extending the empirical model in 19 to account for trading activity in the week or two weeks prior to filing. Indeed, as the table shows, we find that there is a weakly higher trading activity just before filing compared to the rest of the quarter. However, we still find that trading activity increases just after a 13-F filing compared to just before and that the increase in explained entirely by trades with periphery institutions.

6.2.5 Information revelation and market liquidity

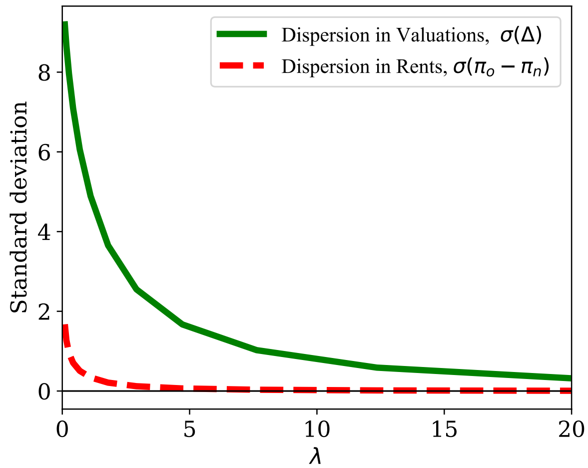
In our theory, screening ability is only relevant as a result of trading frictions that generate dispersion in the valuation of assets, $\Delta(\theta)$, given by equation (7). Under the extreme case where λ is zero, an investor's valuation is driven entirely by their utility flow ν since they have no outside options in trade. Alternatively, under the opposite extreme in which investors can buy and sell assets immediately in a competitive market, the distribution of valuations would collapse to the competitive equilibrium price that is independent of any given investor's utility flow. In this case, screening ability becomes irrelevant.

As trading frictions vanish with $\lambda \to \infty$, investors place less weight on their own flow value and greater weight on the expected gain they receive from buying and then selling assets, or in other words, the gain from intermediation. However, as trading frictions vanish, the gain from intermediation becomes smaller as now all investors can more easily intermediate. We illustrate these effects by way of numerical example in Figure 6. The left panel shows that as λ increases, there is a decrease in the dispersion of the expected gain from intermediation, $\pi_o(\theta) - \pi_n(\theta)$, and as a result also of asset valuations, Δ. This creates a decline in price dispersion and a decline in the value of having the ability to learn the private information of a trade counterparty.

When asset valuations become more condensed, the distortions caused by private informa-

[20] This window represents filing in the week of the deadline. Delays over 45 days occur because the deadline falls on a weekend or holiday.

Figure 6: The effects of trading frictions on dispersion in asset valuations and intermediation rents (left) and the effects of 13-F filing (right).



Notes: We set $r = 0.05$, $\eta = \mu = 1/8$, $\rho = 0.8$, $\xi_0 = 0.5$, $\alpha \sim U[0,1]$, and $\nu \sim U[.5, 3.5]$.

tion are reduced and screening ability becomes less relevant in determining trading outcomes. In turn, we should expect that the impact of revealing private information to the market—for example, through a 13-F filing—is mitigated. We illustrate this result in the right-hand panel of Figure 6. The figure shows the effect in our model when we look at the impact in trade probabilities if one investor were to file a 13-F form, where we set the probability that a 13-F filing is informative to be $\rho = 0.8$. When trading frictions are high, the impact on the probability of trade with the periphery is high. However since the core is comprised of experts with $\alpha = 1$, the impact on trade with them is zero. As λ increases, the impact on the probability of trade with the periphery falls as well as the differential effect between core and periphery. Screening ability becomes less relevant in determining a market's structure when assets are easily traded.

We explore these effects in the market for CDS indexes in Table 4, where we proxy for the extent of trading frictions using trade volume as a measure of market liquidity. We repeat the analysis in 6.2.4 separately across three CDS-index classes with varying degrees of market liquidity. IHS Markit's CDX index class on North American entities is by far the most liquid in terms of trading frequency, accounting for roughly 65% of trades in our sample. The second most liquid class is Markit's CMBX indexes referencing commercial mortgage-backed securities, which accounts for a much smaller fraction of trades (23% of trades). The group "Other" accounts for the remainder and includes indexes that trade infrequently, such as those that reference subprime mortgage backed securities or municipal CDS, swaps referencing interest and principal components of agency pools, or the Dow Jones CDX family.

We find that as an index is traded more frequently in the market, the impact of a 13-F on trade with the periphery is diminished, as well as the effect relative to trade with the core. Using

Table 4: Impact of 13-F filing on trade, by CDX Index Class

	CDX	CMBX	Other	CDX	CMBX	Other
	(1)	(2)	(3)	(4)	(5)	(6)
Percent of Trades	65.6%	23.4%	11%	65.6%	23.4%	11%
	Dependent Variable: Trade with Periphery, β^p					
Filed in prev. week, $F_{i,t-x}$	0.091	0.292**	0.594***	0.053	0.178*	0.353*
	(0.089)	(0.116)	(0.188)	(0.089)	(0.102)	(0.185)
R-squared	0.448	0.317	0.111	0.478	0.483	0.148
	Dependent Variable: Trade with Core, β^c					
Filed in prev. week, $F_{i,t-x}$	0.122	0.064	0.207*	0.037	-0.045	0.034
	(0.077)	(0.072)	(0.123)	(0.074)	(0.068)	(0.122)
R-squared	0.392	0.362	0.105	0.455	0.452	0.135
	Dependent Variable: Difference, $\beta^p - \beta^c$					
Filed in prev. week, $F_{i,t-x}$	-0.031	0.228*	0.388**	0.016	0.223**	0.318*
	(0.092)	(0.121)	(0.189)	(0.091)	(0.111)	(0.187)
R-squared	0.164	0.240	0.068	0.216	0.111	0.187
Fixed Effects						
Week − index	yes	yes	yes	yes	yes	yes
Institution	yes	yes	yes	no	no	no
Institution − quarter	no	no	no	yes	yes	yes
Observations	38,376	102,336	268,632	38,376	102,336	268,632

Notes: Sample includes trades of US credit default swap indexes by regulated institutions or those trading CDS indexes on regulated institutions, that filed a 13-F report at least once in the sample period, 2013Q1-2017Q4. The independent variable, $F_{j,t-1}/Frequency$, is a normalized dummy, where the dummy equals to one if institution j filed a 13-F within the week $t-1$. The two dependent variables are dummies if institution j traded CDS index i in week t with a periphery and core institution, respectively. Test on difference: tests whether the difference in the coefficients is equal to zero. Standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

week-index and institution fixed effects, columns (1)-(3), we find that a 13-F filing increases the probability of trade with the periphery by 9.1% in the market for CDX, 29.2% in the market for CMBX, and 59.4% in the remaining markets for low liquidity indexes. We find a differential impact on trade with the periphery relative to the core of 22.8% and 38.8% in the less liquid markets for CMBX and other indexes, but not in the market for CDX. The results are unchanged if we control for institution-quarter fixed effects, shown in columns (4)-(6).

In summary, Table 4 illustrates that the impact of private information on market structure is more relevant for less liquid markets. If trading delays are large, then experts posses a large gain in intermediating assets.

6.2.6 Discussion of empirical results

In the previous section, we provided evidence of the mechanism through which our model generates endogenous intermediation. In this section, we draw a comparison to the literature on intermediation in OTC markets and ask if other mechanisms could explain our empirical results. We argue that the theory we outline above is the only existing framework that can rationalize our empirical results. It is important to highlight that we do not claim that our mechanism is the only determinant of market structure, but our results suggest it is an empirically relevant one.

We begin by noticing that most theories of intermediation in OTC markets assume complete information about asset valuations. For example, Farboodi, Jarosch, and Shimer (2017), Farboodi, Jarosch, and Menzio (2017) and Hugonnier et al. (2014) have models of complete information. Naturally, these theories predict information disclosure has no impact on the probability of trade. The only way to potentially rationalize the effects of 13-F filings in Table 2 under complete information is if filings impact something other than information about valuations. One such example is if filings affect the meeting technology, for instance if institutions use filings to signal their desire to trade and so are more easily located. In Appendix B.4, we argue that such a model is inconsistent with our empirical results as well. First, filers buy and sell actively before and after filings events (Table 7) and that our baseline results in Table 2 also hold when splitting the sample between buyers and sellers (Table 9). These results are inconsistent with the idea that institutions file to signal that they want to buy or sell. Second, our baseline results also hold if we restrict attention to filing events around the end of the filing window, as determined by the SEC. A model of signaling cannot rationalize this—there should be no strategic motives for filing when there is no choice of whether to delay the filing or not. This test is presented in Table 6.

In terms of theories with private information about private values, the literature on information disclosure has proposed a theory about the effects of filing 13-F and strategic delay. Christoffersen, Danesh, and Musto (2015) argue that institutions primarily delay filing to prevent front-running—a situation in which other investors, upon observing the portfolio shares of the reporting institution, infer their future trades and attempt to execute a trade in the same direction before, obtaining a better price. Institutions have an incentive to delay filing a 13-F so they can execute their trading without competition from front-runners. Table 2 finds that the probability of trade increases in the time period following a 13-F report. However, front-running implies that the probability of trade would decrease in the time period following a report. Hence, it would lead to bias in the opposite direction of our results.

Our empirical results are also likely inconsistent with theories building on private information about common values—such as in the lemon's problem. First empirically, if common value information was driving the effects of 13-F filings then we should expect a filing to on average not only impact the trading activity of the filer, but also to impact trading activity between non-filers. For instance revealing information about the underlying default probability of assets in a CDX index —a common value— should affect all trade for that index and not just the filer's. Our regressions control for this possibility by including week-index fixed effects that capture changes across weeks in trade volume for a given index. Even if common value information disproportionately increases trading with the periphery, our 13-F results state that there is an even larger increase for the institution that filed relative to the rest of the market. Our effects are further strengthened when we include trade activity between non-filers in column 3 of Table 2.[21]

In terms of theory, the literature on common value problems in decentralized asset markets has not studied endogenous intermediation. However, we can ask to what extent does revealing common value information *increase* the probability of trade? To do so, we focus in the effects in a model with pooling equilibria, as would arise in our environment. Pooling equilibria are fundamental to models of uniformed intermediaries, such as the literature following Glosten and Milgrom (1985), or models with random search and bilateral trade, such as Chiu and Koeppl (2015). Consider the case of two asset qualities, good and bad. Uninformed bid prices would either be set high in order to buy both good and bad assets or, if the lemons problem is severe enough, set low to only buy bad assets (or no assets at all). A similar intuition holds for uninformed ask prices. In this case, revealing information about the common value would only increase the probability of trade if the market was initially in a freeze —either only bad assets trade or no assets at all. However in our sample, CDS indexes are actively traded between both core and periphery institutions, and there is no apparent disruption in the market. It seems unlikely that this is what temporarily increases in the probability of trade with the periphery after a filing.[22]

6.3 Intermediation in the CDS-Index Market

We close Section 6 by illustrating that intermediation in the market for CDS indexes reflects the form of intermediation in our environment, described in Section 5. Consistent with the theory, we find that institutions that are most central also act as middlemen by equally buying and selling assets, and are more connected—possessing a larger network of trading partners. We also show that our 13-F results also hold along these other measures of intermediation.

6.3.1 Centrality and middleman activity

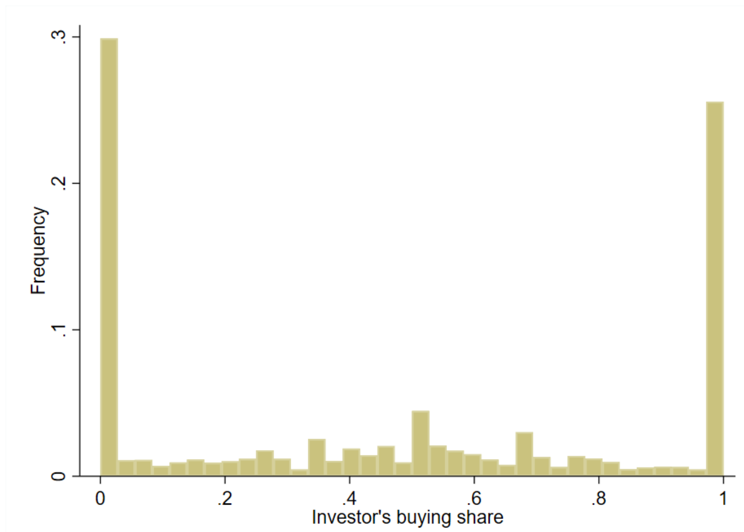
As in Section 5.2, an institutions' role as middlemen is characterized by their buying share. In the CDS data, this corresponds to the share of trades in which an institution buys versus sells protection on a CDS index. Figure 7 presents a histogram for the buying share across institutions in our sample.

The top-5 institutions in terms of volume are all heavily involved in middlemen activity, with buying shares close to 1/2. Among non top-5 institutions, there is substantial heterogeneity in

[21] We also ran a version of the regressions in which we assign non-filers random filing dates from the sampling distribution and examine if a 'fake' filing altered the probability of trade – essentially a placebo test – and found no effects. These results are available upon request.

[22] In a different environment in which investors can post prices and direct their search, a separating equilibrium can emerge, as in Guerrieri et al. (2010) or Guerrieri and Lorenzoni (2011). In equilibrium, investors with high quality assets signal their type by posting high prices and selling with a lower probability. 13-F filings could potentially serve a signalling role outside of the posted price, that could increase the probability of trade. Still, we should expect an increase in the probability of trade for all investors with the same (high quality) asset, which we do not find.

Figure 7: Histogram of institutions' buying share



Notes: Sample includes trades of US CDS indexes by regulated institutions or those trading CDS indexes on regulated institutions in the sample period, 2013Q1-2017Q4. The figure presents the histogram of buying shares, where the buying share of investor *i* is defined as the fraction of trades involving investor *i* where she acquired a CDS.

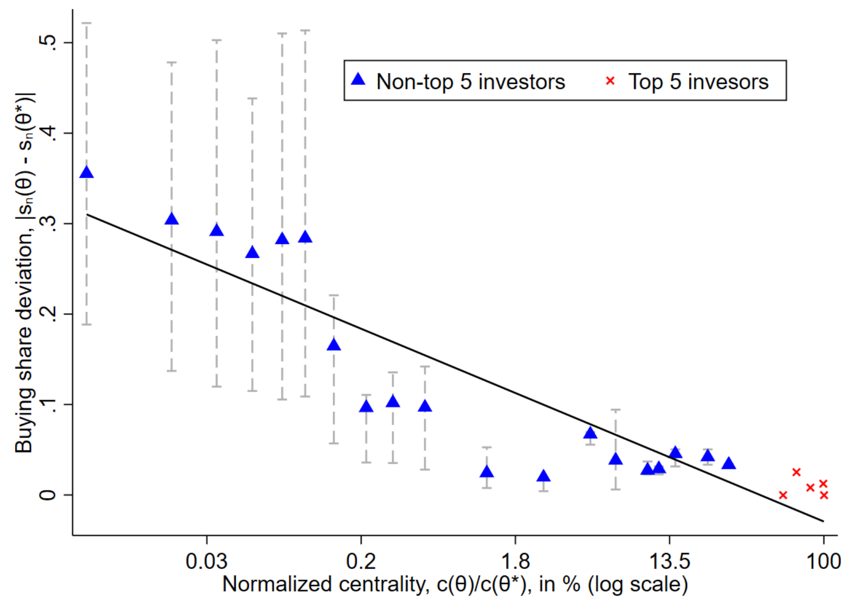
middleman activity. Many non top-5 institutions engage only on one-side of the market. 45.3% of the institutions have buying share below 0.4, and 39.9% of them have buying share over 0.6. However, this also implies that there are many non-top-5 institutions that serve as middlemen, with buying shares between 0.4 and 0.6. This heterogeneity is consistent with our theory that features both central and non-central middlemen.

The theory also provides a connection between an institution's centrality and their propensity to serve as a middleman. Proposition 4 postulates that, for the model with complete information, as an institution's buying share moves away from that of the most central, their centrality falls. And Figure 4 provides numerical examples showing the same result for the model with private information and screening ability heterogeneity. This negative relationship holds true in the CDS-index market, as seem in Figure 8. The figure depicts a scatter plot of the deviation in the buying share of an investor from the buying share of the most central investor (y-axis) and the log of centrality (x-axis). The top-5 institutions are illustrated separately as red crosses. The non-top 5 are binned according to their centrality. The group average is shown as blue triangles and the within-group interquartile range is shown by the dashed-grey lines.

The least central institutions tend to have the largest deviations from the buying share of the most central investor. These institutions also have the greatest heterogeneity in buying versus selling. As centrality increases, institutions tend to buy and sell more equally. Above a centrality threshold all institutions tend to serve as middlemen.

The theory goes one-step further, predicting that experts tend to be high-volume middlemen whereas non-experts tend to be lower-volume middlemen, as can been seen in Figure 4. If so,

Figure 8: Centrality and Buying Share



Notes: Sample includes trades of US CDS indexes by regulated institutions or those trading CDS indexes on regulated institutions in the period of 2013Q1–2017Q4. We compute for each institution the absolute value of her buying share minus the buying share of the most central investor, where the buying share is defined as the fraction trades where the institution is buying a CDS index. The (red) crosses present these values for the top 5 investors. The blue triangles bin these absolute values by centrality of non-top 5 investors, weighted by centrality of the investors. The vertical slashed lines provide the 25th to 75th percentiles of the absolute value of (buying share - buying share of most central investor) for each bin.

we can again use trading activity around 13-F filings to investigate if information revelation has heterogeneous effects on trading with institutions in the sub-sample of middlemen. In Table 5, we report the effect of filing 13-F on trade with middlemen, those institutions with intermediate buying share. In the first two columns, we show the effect on trade with institutions with buying share between 40% and 60%. In the last two columns, we expand the definition of a middleman to buying share between 20% and 80%. The table shows that a 13-F filing increases an institution's probability of trade with periphery middlemen, but has a weaker or zero effect on trade with core middlemen. These results with those in Table 2 suggest that institutions with an informational advantage not only serve as middlemen but also intermediate a high volume of trade, consistent with the predictions of our theory.

6.3.2 Centrality and number of counterparties

Despite that investors in the model are equally likely to meet each other, conditional on meeting they have different probabilities of trade. Through this, our model delivers predictions about the extent of an investor's trading network. In Section 5.3, we showed that screening ability is a force that increases the trading network of investors, either measured as the number of unique counterparties (Proposition 7) or the number of counterparties weighted by trade volume

Table 5: Impact of a 13-F filing on trade with middlemen

	$0.4 \leq$ Buying share ≤ 0.6		$0.2 \leq$ Buying share ≤ 0.8	
	(1)	(2)	(3)	(4)
Trade with Periphery Middlemen, β^p	0.278***	0.184**	0.241***	0.150**
	(0.079)	(0.078)	(0.076)	(0.076)
R-squared	0.161	0.186	0.170	0.191
Trade with Core Middlemen, β^c	0.096*	-0.009	0.096*	-0.010
	(0.054)	(0.053)	(0.054)	(0.053)
R-squared	0.182	0.204	0.182	0.204
Test on difference, $\beta^p - \beta^c$	0.182**	0.193**	0.144*	0.160**
	(0.076)	(0.076)	(0.052)	(0.074)
Fixed Effects				
Week $-$ index	yes	yes	yes	yes
Institution	yes	no	yes	no
Institution $-$ quarter	no	yes	no	yes
Observations	460,512	460,512	460,512	460,512

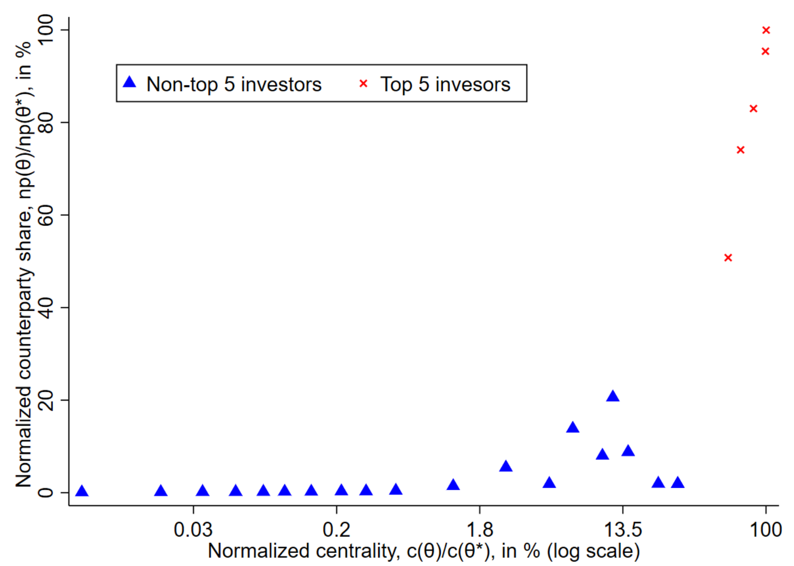
Notes: Sample includes trades of US credit default swap indexes by regulated institutions or those trading CDS indexes on regulated institutions, that filed a 13-F report at least once in the sample period, 2013Q1-2017Q4. The independent variable is a normalized dummy, where the dummy is equal to one if institution j filed a 13-F in the previous week. The two dependent variables are dummies if institution j traded CDS index i in week t with a periphery and core institution, respectively, who have a buying share in $[0.4, 0.6]$ in columns (1)-(2) or $[0.2, 0.8]$ in columns (3)-(4). Test on difference: tests whether the difference in the coefficients is equal to zero. Standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

(Proposition 8). Indeed, experts with $\alpha = 1$ will have the most extensive trading network among all investors measured in either way. Further, unlike in most random search frameworks, some investors will have an incomplete network as a direct result of private information.

Figure 9 depicts the relationship between an institution's centrality, $c(\theta)$, and its number of unique counterparties relative to those of the most central investor, $np(\theta)/np(\theta^*)$. By Proposition 8, our measure of centrality aligns exactly with the measure of connectivity when weighting connections by trade volume. Hence, one can view the x-axis in both figures as either centrality, $c(\theta)$, or strength, $st(\theta)$. The top-5 institutions are illustrated in red crosses while the non-top-5 institutions are binned by centrality and illustrated as blue triangles. The figure shows that top-5 institutions have a significantly larger amount of trade counterparties than institutions in the periphery. Even those institutions just outside the top-5 have a substantially lower number of connections compared to the top-5 institutions. Given our 13-F results that suggest the top-5 have an advantage in screening ability, these results also suggest that this advantage leads these institutions to trade with an extensive group of counterparties.

A potential concern is that a mechanical effect explain the results in Figure 9: When an institution trades more, they may naturally trade with more counterparties. By way of extreme example, if an institution only trades once then it can have at most one unique counterparty. If it trades twice, it can have at most two counterparties, and so forth. Figure 10 controls for this

Figure 9: Centrality, $c(\theta)$, and number of counterparties, $np(\theta)$



Notes: Sample includes trades of US credit default swap indexes by regulated institutions or those trading CDS indexes on regulated institutions in the sample period, 2013Q1-2017Q4. For each investor we compute the share of trade counterparties, relative to the share of trade counterparties of the most central investor. We then bin these shares, as a function of the normalized centrality of the investor. The (red) crosses present these values for the top 5 investors. The blue triangles bin the shares by the centrality of non-top 5 investors, weighted by centrality of the investors.

by ranking trades by the centrality of their institution, then binning such that each bin present in the figure has the same number of trades. Hence, the max possible number of counterparties in each group are equal. The same strong positive relationship between the number of trade counterparties and centrality remains.

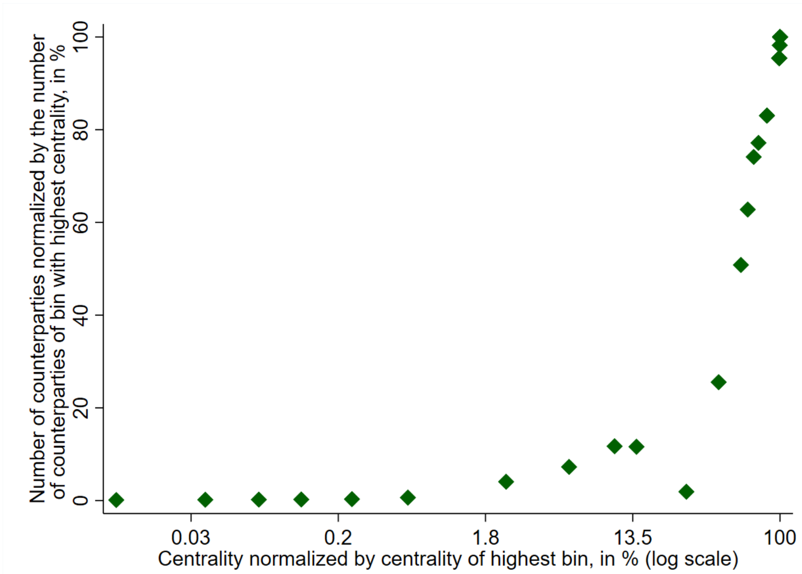
6.3.3 Discussion of results on intermediation

While in Section 6.2 we provided evidence supporting the prevalence of private information in determining trade outcomes, Section 6.3 connects our theoretical findings on market structure with their empirical counterparts.

Section 6.3.1 confirmed empirically an important prediction of the model: the most central investors (in terms of trade volume) are highly engaged in middleman activity, and middleman activity falls with the investor's centrality. These predictions cannot be supported in random search models of market structure built under complete information. There, by construction, an investor's buying share is equal to 0.5, independently of their share of aggregate trade volume. We further augment these findings by showing that the effect of private information on trade outcomes runs through trade volume.

Section 6.3.2 presented evidence regarding the connection between private information, centrality, and the size of the trade network of an investor, $np(\theta)$. Consistent with the theory, trading in CDS data shows that core investors—which are experts in our model—exhibit a considerably

Figure 10: Centrality, $c(\theta)$, and number of counterparties, $np(\theta)$, when bins have same number of trades



Notes: Sample includes trades of US credit default swap indexes by regulated institutions or those trading CDS indexes on regulated institutions in the sample period, 2013Q1-2017Q4. We bin trades by the centrality of the investor, and we compute the share of unique counterparties for trades in a bin, relative to the share of unique counterparties of the bin with highest centrality. Because all trades of an investor have the same centrality, there is no unique way to bin them. Therefore, we repeat the procedure described above 1,000 times, and we take averages across trials.

larger trade network than periphery investors, even controlling by the number of trades. Again, this prediction cannot be supported by random search models of market structure building on complete information, as in these models the trade network is complete for all investors.

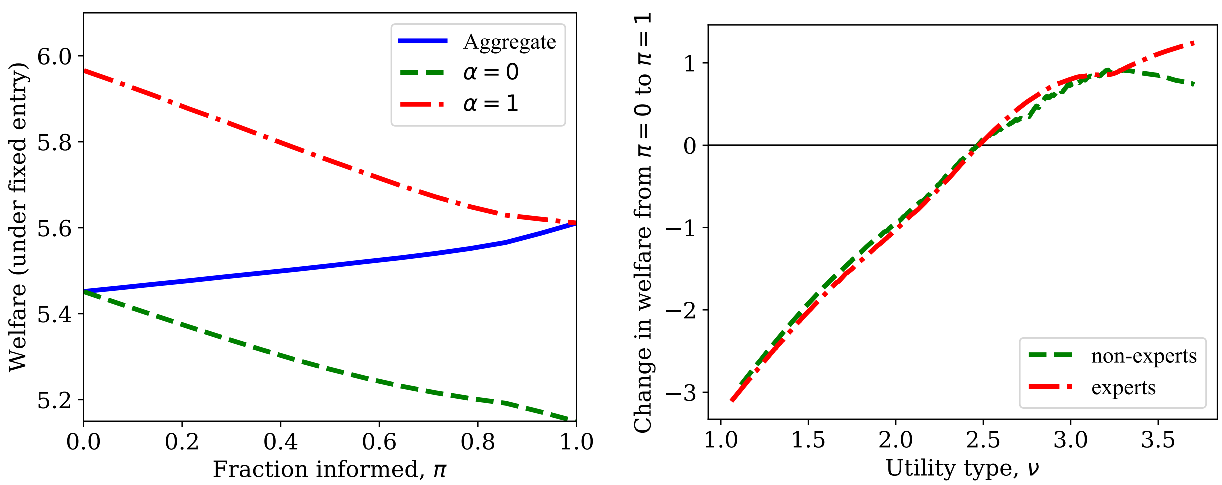
Overall, Sections 6.2 and 6.3 provided strong evidence of the relevance of private information in explaining trade outcomes in OTC markets, and its relevance in shaping up several different dimensions of the core-periphery notion of market structure.

7 The efficient amount of screening

Is it always better to have more experts in the market? On the one hand, since investors with high screening ability trade more efficiently, more experts imply a faster reallocation of assets from low- to high-valuation investors. On the other hand, because investors with high screening ability extract more rents when trading, more experts imply a redistribution of the gains from trade towards experts. While a faster reallocation of assets from low- to high-valuation investors tend to always increase welfare, a redistribution of the gains from trade towards experts can increase or decrease welfare and distort the incentives to participate in the market. With this in mind, in this section we study how expertise affects the size of the market and welfare of market participants.

To capture market size, we consider a variation of our model with an entry decision. Assume that screening ability, α, is either 0 or 1, and let $\pi \in [0,1]$ be the fraction of investors with $\alpha = 1$, which we will call *experts*. As before, the types θ is drawn from a distribution $F(\theta)$ with density $f(\theta)$.[23] At the beginning of time after realizing their type, $\theta = (\alpha, \nu)$, but before the initial allocation of assets, all investors must make an entry decision by paying a fixed utility cost $\kappa \geq 0$. Let the distribution of active investors, those who decided to pay the cost κ, be $\hat{f}(\theta)$. Given $\hat{f}(\theta)$, the lifetime expected utility of a single investor in the stationary equilibrium is $\bar{V}(\theta) = [\hat{\phi}_o(\theta)V_o(\theta) + \hat{\phi}_n(\theta)V_n(\theta)]/\hat{f}(\theta)$, where $\hat{\phi}_o$ and $\hat{\phi}_n$ are densities of a stationary equilibrium, given $\hat{f}$. Investors of type θ become active if $\bar{V}(\theta) \geq \kappa$, otherwise they remain idle. In any stationary equilibrium $\bar{V}(\theta) = \bar{V}(\alpha, \nu)$ is increasing in ν. Therefore, there is a cutoff rule for entry such that all investors with screening ability α enter if $\nu \geq \nu^*_\alpha$. Additionally, since $\bar{V}(1,\nu) \geq \bar{V}(0,\nu)$, we must have that $\nu^*_1 \leq \nu^*_0$.

Figure 11: The effects of information on welfare under fixed entry and $\xi_o = 0$.



Notes: The example above assume $\alpha \in \{0,1\}$, where $\pi \in [0,1]$ is the fraction of experts $\alpha = 1$. The remaining parameters are $r = 0.05, \mu = \eta = 1/8, \lambda = 4$, and $\nu \sim U[1.0, 3.5]$.

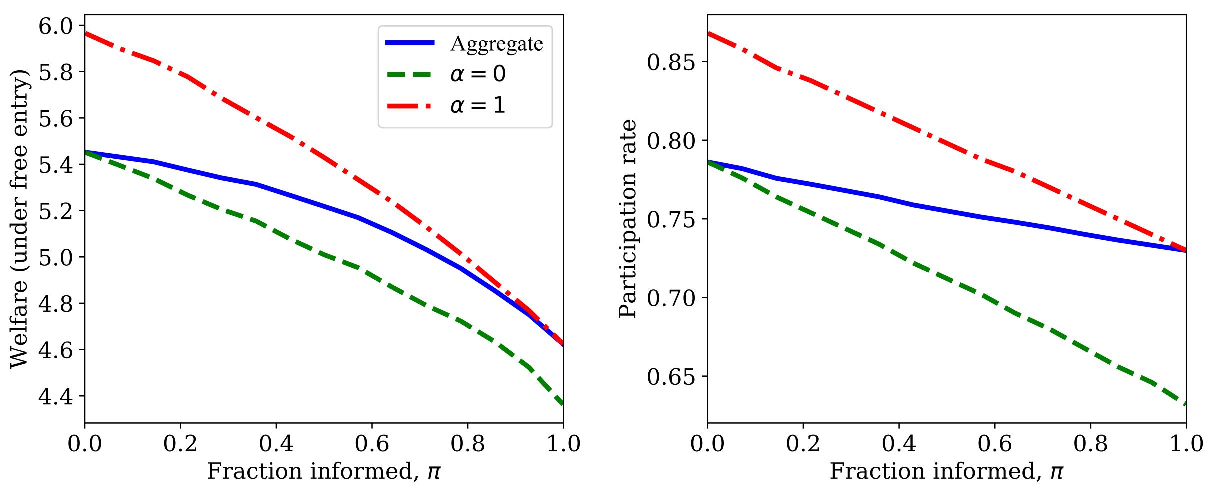
What are the effects of increasing the fraction of experts, π, in this environment? First, consider the effects in the baseline model with no entry. The left panel of Figure 11 illustrates the impact on aggregate welfare (solid-blue line), as well as on the welfare of experts (dashed-dotted-red line) and those with no expertise (dotted-green line). For this example, we assume buyers set the terms of trade $\xi_o = 0$ and set $r = 0.05$, $\mu = \eta = 1/8$, $\lambda = 4.0$, and $\nu \sim U[1.0, 3.5]$.

Increasing the number of experts increases aggregate welfare, however it lowers welfare for both experts and non-experts on average. Why? The right panel illustrates the change in welfare for any given investor of type ν and α as a result of going from an environment with no experts

[23] To prevent asset issuance from playing an important role, we assume that the support of ν associated with F is a subset of $\mathbb{R}_{++}$. In this case, every investor issues an asset when they have an opportunity.

$\pi = 0$ to one with all experts $\pi = 1$.[24] Increasing expertise reallocates the gains from trade from those with low value to those with high. Low value investors are natural sellers and, since buyers set the terms of trade their welfare heavily relies on informational rents. By increasing the measure of experts, informational rents are destroyed and the welfare for low-value investors falls. This occurs for both non-experts and experts since an investor's expertise is irrelevant when their counterparty sets the terms of trade.

Figure 12: The effects of information on welfare and participation with free entry and $\xi_o = 0$.



Notes: The example above assume $\alpha \in \{0, 1\}$, where $\pi \in [0, 1]$ is the fraction of experts $\alpha = 1$. The remaining parameters are $r = 0.1, \mu = \eta = 1/8, \lambda = 4$, and $\nu \sim U[0.5, 3.5]$.

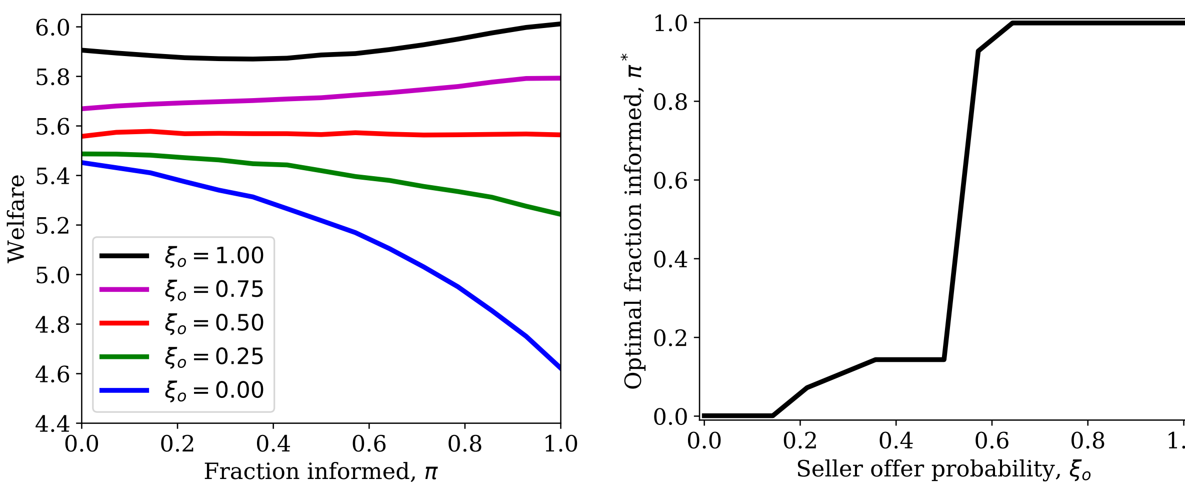
Under free entry, the reduction in informational rents reduces participation of low value investors. Figure 12 illustrates the impact on welfare (left panel) and the participation rate (right panel) as the fraction of experts increases. Participation falls for both experts and non-experts, and even though the compositional effect serves to increase participation, aggregate participation falls. When the participation effect is strong enough, as in this example, aggregate welfare falls even though information and allocational efficiency improve. Incorporating a free-entry decision in the model introduces a hold-up problem. When ξ_o is low, low-value investors do not internalize the social benefit of entering the market and participation is depressed. When the fraction of experts is low, the informational rents accrued by these agents partially compensates for a low bargaining share and leads to more participation.

As the probability of setting the terms of trade shifts from buyers to sellers, low value investors rely less on informational rents and, as a result, increasing information can potentially lead to gains in participation and welfare. In the left panel of Figure 13, we illustrate the impact on aggregate welfare from increasing expertise for various values of $\xi_o \in [0, 1]$. Welfare can be

[24]Numerically, we set π equal to 0.0001 and 0.9999.

increasing, decreasing, humped- or u-shaped in the fraction of informed investors. The right panel of Figure 13 shows the welfare maximizing fraction of experts π as ξ_o ranges from zero to one. If a constrained planner can set level of expertise, π, then it is optimal have few experts when buyers set the terms of trade in order to improve participation through informational rents. However when sellers set the terms of trade, the opposite is true and the planner wants to increase expertise to the maximum. For intermediate value of ξ_o, the planner optimally chooses an intermediate value of expertise.

Figure 13: The effects of information on welfare under free entry for $\xi_o \in [0, 1]$.



Notes: The examples above assume $\alpha \in \{0, 1\}$, where $\pi \in [0, 1]$ is the fraction of experts $\alpha = 1$ and owners make the offer with probability ξ_o. The remaining parameters are $r = 0.03$, $\mu = \eta = 1/10$, $\lambda = 6$, and $\nu \sim N(1, 25)$.

8 Concluding remarks

When trading in a bilateral meeting, investors must gauge the willingness to pay of their trade counterparties. Differential ability in gauging this information, which we call screening expertise, can affect the functioning and structure of decentralized markets. We propose a theory of financial intermediation based on heterogeneity in the information investors possess about the trading motives of their counterparties and their willingness to pay. Superior information allows investors to avoid distortive mechanisms and, as a result, the investors who are the most central in trade must be endowed with superior information.

Heterogeneity in screening expertise has important implications for trade outcomes and market structure that we explore empirically. Regarding trade outcomes, we provide empirical evidence in line with this central prediction of our theory by examining the effect of filing a 13-F form to the SEC, which makes public the institution's holdings of SEC regulated securities public information, on the probability of trade in the CDS index market. We show that a 13-F filing increases the probability of trade with periphery institutions to a greater extent than with core institutions, and in several specifications we find no effect of a 13-F filing on trade with core institutions. Regarding market structure, we provide empirical evidence using the CDS data that the extent of middleman activity and trade network depth of investors are consistent with the predictions of our model. While the trade outcome and market structure predictions follow from our theory, they do not follow from other theories of financial market intermediation based on complete information, private information about common values, or contact rate heterogeneity. Further, we show that better information, in the sense of providing higher screening ability to all investors, is not always desirable in OTC markets with intermediation.

In recent years, many important theoretical contributions have been advanced with the goal of understanding the driving forces of intermediation in OTC markets. Our paper provides a new theory of intermediation along with empirical support using micro-data that suggests the mechanism we study is relevant. Since these other theories are built on the assumption of complete information, our empirical results should not be seen as a test of these models since they do not provide theoretical predictions about information revelation and market structure. However, we also emphasize that our results imply that information heterogeneity is an important determinant of which institutions populate the core of OTC markets. To our knowledge, there is no existing empirical work that tests other theories of endogenous intermediation nor are there existing theories about private information and endogenous intermediation in OTC markets.

Asymmetric information regarding the willingness to pay of investors is a feature of most, if not all, OTC markets. Should policy-makers worry about the lack of full information? Our numerical explorations suggest that the answer depends on the specifics of the particular market. In markets where buyers set the terms of trade, full opacity about trade motives is strictly preferred to full information. The result follows given that although private information distorts trade, it also provides a sharing of the trade surplus among the two investors in a bilateral meeting. And sharing the trade surplus redistributes gains from buyers to sellers, motivating low valuation investors to enter the market, and help in the process of intermediation.

How important are private values and heterogeneous information in OTC markets? In this paper we showed that we can build a theory of financial intermediation relying on them, and provided novel evidence that they both shape up market structure in the decentralized trading network of Credit Default Swaps. We leave for future research to quantify the extent of private values and heterogeneous information in decentralized markets. We believe that our paper provides an abundance of statistics that can be used for this.

In the paper we study the role of information in a stationary economy, but economies are in constant change. Aggregate shocks, such as the 2008 financial crisis and the 2020 Covid-19 pandemic, can impact the distribution of asset valuations—for example, by affecting the liquidity needs of market participants. Our model provides a natural mechanism through which changes in the distribution of valuations affects asset misallocation through pricing. As a concrete example, during the 2008 crisis when house prices collapsed and mortgage rates were low, some homeowners were under water and forced to sell while others found themselves in a good position to buy. To what extent was asymmetric information detrimental to the housing market recovery by preventing efficient trade? Further, our model predicts that intermediaries are experts who price efficiently. What role do expert intermediaries have in asset market dynamics, and, in particular, to the housing market recovery? Our results suggest that information plays an important role, but we need more research to quantify its importance and the mechanism in place after aggregate shocks.

Aggregate shocks can also impact market participation, the amount of expertise, and the incentives to intermediate assets, in a similar fashion to what we study in Section 7. Not only is intermediation in OTC markets important during normal times, but especially so after aggregate shocks. For example, the COVID-19 pandemic triggered investors demand to trade in the US treasury and corporate bond markets, overwhelming dealers capacity to intermediate the market and causing yields to rise sharply until the Federal Reserve stepped in.[25] Do experts enter or exit the market, and how does this interact with the speed of recovery? Our empirical evidence suggests it is important to study the role of intermediation in these episodes through the lens of how expertise is impacted.

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[25]For evidence in the Treasury market see Duffie (2020) and for the corporate bond market see Kargar, Lester, Lindsay, Liu, Weill, and Zuniga (2020).

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A Appendix: Proofs

A.1 Price distortions

In this subsection we provide the proofs for the results in sub-section 3.1. Specifically, we provide the proofs of Lemma 1 and Corollary 1, which show how trade is distorted under private information when the owner makes the take-it-or-leave-it offer. The proofs of Lemma 2 and Corollary 2 are analogous and we omit them here.

Proof of Lemma 1. Note that $obj_o(ask;\alpha_n)$ is smaller or equal to zero for any ask smaller or equal to Δ_o. Since $M_n(\cdot;\alpha_n)$ is a cumulative distribution, and therefore right continuous, and $M_n(\Delta_o;\alpha_n) < 1$, there exists $\hat{ask} > \Delta_o$ such that $M_n(\hat{ask};\alpha_n) < 1$. As a result, we have that $obj_o(ask_o;\alpha_n) \geq obj_o(\hat{ask};\alpha_n) = [\hat{ask} - \Delta_o][1 - M_n(\hat{ask};\alpha_n)] > 0 \Longrightarrow ask_o \not\leq \Delta_o$.

Proof of Corollary 1. Because $1 - M_n(\Delta_o;\alpha_n) \geq M_n(\Delta_o + \bar{\epsilon}/2;\alpha_n) - M_n(\Delta_o;\alpha_n) > 0$, Lemma 1 implies that $ask_o - \Delta_o > 0$. Let $\epsilon = ask_o - \Delta_o - min\{ask_o - \Delta_o, \bar{\epsilon}\}/2 \in (0, \bar{\epsilon})$. Then have that $M_n(ask_o - \epsilon;\alpha_n) - M_n(\Delta_o;\alpha_n) = M_n(\Delta_o + min\{ask_o - \Delta_o, \bar{\epsilon}\}/2) - M_n(\Delta_o;\alpha_n) > 0$.

A.2 Optimal Mechanisms

In this subsection we characterize the optimal selling/buying mechanism, which is going to be used in our equilibrium existence proof. It is optimal for the owner/non-owner to use an ask/bid price to sell/buy the asset when the investor does not observe the type of the counterpart in a meeting. We use this characterization when proving existence of an equilibrium. It is worth mentioning that many variations of the techniques and results we discuss in this section are standard in the mechanism design literature, and can be found in, for example, textbooks such as Mas-Colell, Whinston, Green, et al. (1995). They are included in this section for completeness of the manuscript.

A.2.1 The optimal selling mechanism as an ask price

Assume that the distribution of reservation values of non-owners, $M_n(\cdot;\alpha_n)$, has non-empty support $[\underline{\Delta}, \bar{\Delta}]$ and density $m_n(\cdot;\alpha_n)$ that is bounded above and away from zero. We omit the argument α_n from $M_n(\cdot;\alpha_n)$ and $m_n(\cdot;\alpha_n)$ to keep the notation short.

We apply the revelation principle and focus on direct mechanisms. For an owner with reservation value Δ_o, a direct mechanism is a pair $(p, x) : [\underline{\Delta}, \bar{\Delta}] \to [0, 1] \times \mathbb{R}$, where $p(\Delta_n)$ is the probability of transferring the asset from the owner to a non-owner with reservation value Δ_n, and $x(\Delta_n)$ is the transfer from the non-owner to the owner.

The problem of an owner with reservation value Δ_o is

$$\max_{p,x} \int \left[x(\Delta_n) - p(\Delta_n)\Delta_o\right] m_n(\Delta_n) d\Delta_n \tag{20}$$

subject to

$$IR: \quad p(\Delta_n)\Delta_n - x(\Delta_n); \geq 0 \text{ and} \tag{21}$$

$$IC: \quad p(\Delta_n)\Delta_n - x(\Delta_n) \geq p(\hat{\Delta}_n)\Delta_n - x(\hat{\Delta}_n); \tag{22}$$

for all Δ_n and $\hat{\Delta}_n$ in the support $[\underline{\Delta}, \bar{\Delta}]$.

We will show a solution to (20)-(22) is associated with an ask price. To do so, it is helpful to start with the following lemmas.

Lemma 7. *A mechanism (p, x) satisfies (21) and (22) if, and only if, $p(\Delta_n)$ is non-decreasing and $U_n(\Delta_n) := p(\Delta_n)\Delta_n - x(\Delta_n) = U_n(\underline{\Delta}) + \int_{\underline{\Delta}}^{\Delta_n} p(\Delta)d\Delta$ with $U_n(\underline{\Delta}) \geq 0$.*

Proof. Let us start showing the necessity part. By reorganizing the IC constraint (22) we can show that $p(\Delta_n)[\Delta_n - \hat{\Delta}_n] \geq U(\Delta_n) - U(\hat{\Delta}_n) \geq p(\hat{\Delta}_n)[\Delta_n - \hat{\Delta}_n]$ for any $\Delta_n > \hat{\Delta}_n$. So we can conclude that p is non-decreasing. Now we can also apply the envelope theorem in Milgrom and Segal (2002) (see sub-section 3.1) to conclude that

$$U(\Delta_n) = U(\underline{\Delta}) + \int_{\underline{\Delta}}^{\Delta_n} p(\Delta)d\Delta.$$

Since (p, x) satisfies (21), $U(\Delta_n) \geq 0$ for all Δ_n and we have that $U(\underline{\Delta}) \geq 0$.

For the sufficient part, if $U(\Delta_n) = U(\underline{\Delta}) + \int_{\underline{\Delta}}^{\Delta_n} p(\Delta)d\Delta$ and $U(\underline{\Delta}) \geq 0$, then $U(\Delta_n) \geq 0$ for all Δ_n since $p(\Delta_n) \in [0, 1]$. Hence, the mechanism satisfies (21). Moreover,

$$U(\Delta_n) - U(\hat{\Delta}_n) = \int_{\hat{\Delta}_n}^{\Delta_n} p(\Delta)d\Delta_n.$$

And because p is non-decreasing, we have that $U(\Delta_n) \geq U(\hat{\Delta}_n) + p(\hat{\Delta}_n)[\Delta_n - \hat{\Delta}_n] \implies U(\Delta_n) \geq p(\hat{\Delta}_n)\Delta_n - x(\hat{\Delta}_n)$. That is, the IC constraint (22) is satisfied. □

Lemma 8. *Let the distribution of non-owners, M_n, have a non-empty support $[\underline{\Delta}, \bar{\Delta}]$ and a density m_n that is bounded above and away from zero. A direct mechanism (p^*, x^*) solves problem (20) if, and only if, it solves problem*

$$\max_{p,x} \int p(\Delta_n) \left[\Delta_n - \frac{1 - M_n(\Delta_n)}{m_n(\Delta_n)} - \Delta_o\right] m_n(\Delta_n) d\Delta_n \tag{23}$$

subject to $p(\Delta_n)$ being increasing and x satisfying $U(\Delta_n) := p(\Delta_n)\Delta_n - x(\Delta_n) = \int_{\underline{\Delta}}^{\Delta_n} p(\Delta)d\Delta$.

Proof. Using Lemma 7, we can rewrite the objective function given by problem (20) as

$$\int \left[x(\Delta_n) - p(\Delta_n)\Delta_o\right] m_n(\Delta_n) d\Delta_n =$$

$$\int p(\Delta_n) \left[\Delta_n - \Delta_o\right] m_n(\Delta_n) d\Delta_n - \int \int_{\underline{\Delta}}^{\Delta_n} p(\Delta)d\Delta m_n(\Delta_n) d\Delta_n - U(\underline{\Delta}).$$

We apply integration by parts to obtain that

$$\int_{\underline{\Delta}}^{\bar{\Delta}} \int_{\underline{\Delta}}^{\Delta_n} p(\Delta) d\Delta m_n(\Delta_n) d\Delta_n = \int p(\Delta_n)[1 - M_n(\Delta_n)] d\Delta_n.$$

Combining the above equations, we have that the objective function is

$$\int p(\Delta_n) \left[\Delta_n - \frac{1 - M_n(\Delta_n)}{m_n(\Delta_n)} - \Delta_o \right] m_n(\Delta_n) d\Delta_n - U(\underline{\Delta}).$$

$U(\underline{\Delta})$ is set to zero to maximize profits and, by Lemma 9, $p(\Delta_n)$ is non-decreasing. □

Now we can characterize the optimal selling mechanism.

Proposition 11. *Let the distribution of non-owners, M_n, have a non-empty support $[\underline{\Delta}, \bar{\Delta}]$ and a density m_n that is bounded above and away from zero. Define the functions*

$$\bar{H}_n(q) = \min_{\substack{\omega, r_1, r_2 \\ \omega r_1 + (1-\omega) r_2 = q}} \{\omega H_n(r_1) + (1-\omega) H_n(r_2)\} \quad \text{and} \quad \bar{h}_n(q) = \frac{d\bar{H}_n(q)}{dq}, \tag{24}$$

where $h_n(q) = M_n^{-1}(q) - \frac{1-q}{m_n(M_n^{-1}(q))}$ and $H_n(q) = \int_0^q h_n(r) dr$; and

$$c_n(\Delta) = \bar{h}_n\left(M_n(\Delta)\right) \quad \text{and} \quad c_n^{-1}(\Delta) = \inf\{\Delta_n \in [\underline{\Delta}, \bar{\Delta}]; c_n(\Delta_n) \geq \Delta\}.$$

The direct mechanism

$$\left(p(\Delta_n), x(\Delta_n)\right) = \begin{cases} \left(1, c_n^{-1}(\Delta_o)\right) & \text{if } c_n(\Delta_n) \geq \Delta_o \\ (0, 0) & \text{otherwise} \end{cases}$$

achieves the maximum in problem (20). Moreover, a take-it-or-leave-it offer with bidding price of ask $=$ $c_n^{-1}(\Delta_o)$ decentralizes the above mechanism.

Proof. We can write the objective function as

$$\int p(\Delta_n) \left[\Delta_n - \frac{1 - M_n(\Delta_n)}{m_n(\Delta_n)} - \Delta_o \right] dM_n = \int p(\Delta_n) \left[h_n(M_n(\Delta_n)) - \Delta_o \right] dM_n$$
$$= \int p(\Delta_n) \left[c_n(\Delta_n) - \Delta_o \right] dM_n + \int p(\Delta_n) \left[h_n(M_n(\Delta_n)) - \bar{h}_n(M_n(\Delta_n)) \right] dM_n.$$

Let us consider the last term of the above equation.

$$\int p(\Delta_n) \left[h_n(M_n(\Delta_n)) - \bar{h}_n(M_n(\Delta_n)) \right] dM_n$$
$$= p(\Delta_n) \left[H_n(M_n(\Delta_n)) - \bar{H}_n(M_n(\Delta_n)) \right]_{\underline{\Delta}}^{\bar{\Delta}} - \int \left[H_n(M_n(\Delta_n)) - \bar{H}_n(M_n(\Delta_n)) \right] dp(\Delta_n).$$

Since $\bar{H}_n$ is the convex-hull of H_n, they coincide at the boundary points $\underline{\Delta}$ and $\bar{\Delta}$, and we conclude that the first term of the final expression is equal to 0. The objective function equals

$$\int p(\Delta_n) \left[c_n(\Delta_n) - \Delta_o \right] dM_n - \int \left[H_n(M_n(\Delta_n)) - \bar{H}_n(M_n(\Delta_n)) \right] dp(\Delta_n).$$

It is easy to see that our proposed mechanism maximizes the first term since, by construction, $p(\Delta_n) = 1$ whenever $c_n(\Delta_n) \geq \Delta_o$. Also, the proposed mechanism maximizes the second term.

To see this, note that the second term is nonpositive for any weakly increasing $p(\Delta_n)$. In our proposed mechanism, this term is exactly zero because whenever $H_n(M_n(\Delta_n)) - \bar{H}_n(M_n(\Delta_n)) > 0$ the derivative $g(q) = \frac{\bar{H}_n G(q)}{dq}$ is constant due the convex hull and, as a result, $dp(\Delta_n)$ is zero. Thus, the proposed mechanism achieves the maximum in problem (23) and, therefore, in problem (20). □

For the owner, asking the price $c_n^{-1}(\Delta_o)$ and selling the asset whenever the non-owner has a reservation value higher than the ask price is an optimal mechanism. Therefore, it coincides with the optimal ask price analyzed in Section 3.

A.2.2 The optimal buying mechanism as a bid price

The problem of a non-owner is analogous to the problem of an owner presented in subsection A.2.1, and, to keep the presentation short, here we will only formulate the problem and present the main result.

We assume that the distribution of reservation values of owners, $M_o(\cdot;\alpha_o)$, has a non-empty support $[\underline{\Delta}, \bar{\Delta}]$ and a density $m_o(\cdot;\alpha_o)$ which is bounded above and away from zero, and omit the argument α_o from $M_o(\cdot;\alpha_o)$ and $m_o(\cdot;\alpha_o)$ to keep the notation short. The problem of a non-owner with reservation value Δ_n is given by

$$\max_{p,x} \int \left[p(\Delta_o)\Delta_n - x(\Delta_o)\right] m_o(\Delta_o) d\Delta_o \tag{25}$$

subject to

$$IR: \quad x(\Delta_o) - p(\Delta_o)\Delta_o \geq 0 \text{ and} \tag{26}$$

$$IC: \quad x(\Delta_o) - p(\Delta_o)\Delta_o \geq x(\hat{\Delta}_o) - p(\hat{\Delta}_o)\Delta_o; \tag{27}$$

for all Δ_o and $\hat{\Delta}_o$ in the support $[\underline{\Delta}, \bar{\Delta}]$. We now characterize mechanism that are IC and individually rational, that is, satisfy equations (26) and (27).

Lemma 9. *A mechanism (p, x) satisfies (26) and (27) if, and only if, $p(\Delta_o)$ is non-increasing and $U_o(\Delta_o) := x(\Delta_o) - p(\Delta_o)\Delta_o = U_o(\bar{\Delta}) + \int_{\Delta_o}^{\bar{\Delta}} p(\Delta)d\Delta$ with $U_o(\underline{\Delta}) \geq 0$.*

We are now ready to state our characterization of the optimal buying mechanism.

Proposition 12. *Let the distribution of owners, M_o, have a non-empty support $[\underline{\Delta}, \bar{\Delta}]$ and a density m_o that is essentially bounded above and away from zero. Define the functions*

$$\bar{H}_o(q) = \min_{\substack{\omega, r_1, r_2 \\ \omega r_1 + (1-\omega) r_2 = q}} \{\omega H_o(r_1) + (1-\omega) H_o(r_2)\}, \quad \bar{h}_o(q) = \frac{d\bar{H}_o(q)}{dq} \tag{28}$$

where $h_o(q) = M_o^{-1}(q) + \frac{q}{m_o(M_o^{-1}(q))}$ and $H_o(q) = \int_0^q h_o(r)dr$, and

$$c_o(\Delta) = \bar{h}_o\left(M_o(\Delta)\right) \quad \textit{and} \quad c_o^{-1}(\Delta) = \sup\{\Delta_o \in [\underline{\Delta}, \bar{\Delta}]; c_o(\Delta_o) \leq \Delta\}.$$

The direct mechanism

$$\big(p(\Delta_o), x(\Delta_o)\big) = \begin{cases} \big(1, c_o^{-1}(\Delta_n)\big) & \textit{if } c_o(\Delta_o) \leq \Delta_n \\ (0,0) & \textit{otherwise} \end{cases}$$

achieves the maximum in problem (25). *Moreover, a take-it-or-leave-it offer with bidding price of bid* $=$ $c_o^{-1}(\Delta_n)$ *decentralizes the above mechanism.*

For the non-owner, biding the price $c_o^{-1}(\Delta_n)$ and buying the asset whenever the owner has a reservation value lower than the bid price is an optimal mechanism. Therefore, it coincides with the optimal bid price analyzed in Section 3.

A.3 Equilibrium

There exists a symmetric steady-state equilibrium, with bid and ask prices associated with optimal buying and selling mechanisms. The strategy for our proof is the following. We define an operator T mapping the reservation value and distribution of types across owners, Δ and Φ, into a new reservation value and distribution, $\hat{\Delta}$ and $\hat{\Phi}$, using the equilibrium conditions. Such procedure may lead to functions that are not differentiable. This can be a problem because bounded and closed subsets of the space of continuous functions is not compact. To account for that, we show that the operator maps Lipschitz continuous functions in Lipschitz continuous functions with the same constant. Since bounded and closed subsets of the space of Lipschitz continuous functions with the same constant is compact, we can apply Schauder fixed point theorem. We then build the other equilibrium objects from the fixed point we established existence.

Proof of Proposition 1. We first show existence in a truncated economy and later take the limit so it converges to our original economy. Consider a truncation of our economy with preference types $\nu \in [\underline{\nu}, \bar{\nu}]$, where $\bar{\nu} > 0$ is some large constant and $\underline{\nu} < -\lambda\bar{\nu}$. With slight abuse of notation, we use F and f below to denote the cumulative distribution and density of $\theta = (\alpha, \nu)$ truncated in the set $\Theta_M = \Theta \cap [0,1] \times [\underline{\nu}, \bar{\nu}]$. We first show that an equilibrium for this truncated economy exists. Then we take the limit when $\underline{\nu}$ and $\bar{\nu}$ go to infinity and argue for the convergence to an equilibrium of the original economy.

Defining the compact set $\mathcal{E}$: We start defining the space of functions in which we will apply our fixed point theorem. Define first the objects $a = \frac{\lambda}{\lambda+\mu+\eta+r}$, $b = \frac{1}{\lambda+\mu+\eta+r}$, $\kappa = \frac{1}{1-a}$, $\underline{\kappa} = \frac{\eta}{\lambda+\mu+\eta}$, $\bar{\kappa} = \frac{\lambda+\eta}{\lambda+\mu+\eta}$, $\underline{\Delta} = \frac{\underline{\nu}}{r}$, $\bar{\Delta} = \frac{\bar{\nu}}{r}$, $\phi(\theta) = \partial\Phi/\partial\nu$, and $\Delta_\nu = \partial\Delta/\partial\nu$. Let $\mathcal{E}$ be the set of $(\Delta, \Phi) \in \mathcal{C}^0(\Theta_M) \times \mathcal{C}^0(\Theta_M)$ satisfying the following conditions:

$$\Delta(\alpha, 0) \geq 0; \tag{29}$$

$$\underline{\Delta} \leq \Delta(\alpha, \nu) \leq \bar{\Delta}; \tag{30}$$

$$b \leq \frac{\Delta(\alpha,\hat{v}) - \Delta(\alpha,v)}{\hat{v} - v} \leq (1+\kappa)b; \tag{31}$$

$$0 \leq \Phi(\alpha,v) \leq \bar{\kappa}F(\alpha,v); \text{ and} \tag{32}$$

$$\underline{\kappa}\frac{F(\alpha,\hat{v}) - F(\alpha,v)}{\hat{v} - v} \leq \frac{\Phi(\alpha,\hat{v}) - \Phi(\alpha,v)}{\hat{v} - v} \leq \bar{\kappa}\frac{F(\alpha,\hat{v}) - F(\alpha,v)}{\hat{v} - v}. \tag{33}$$

Equations (31) and (33) imply that $\mathcal{E}$ is a compact and convex subspace of $\mathcal{C}^0(\Theta_M) \times \mathcal{C}^0(\Theta_M)$. To see this result, note that $\mathcal{E}$ is uniformly bounded, and Lipschitz continuous with the same constant. Since all inequalities in equations (29)-(33) are weak inequalities, $\mathcal{E}$ is also closed. The Arzelà-Ascoli theorem then implies that any sequence in $\mathcal{E}$ has a converging sub-sequence and we can conclude that $\mathcal{E}$ is compact. The convexity of $\mathcal{E}$ derives from the linearity of inequalities (29) and (33) on (Δ,Φ).

Defining the operator $T : \mathcal{E} \to \mathcal{E}$

We define T in two steps. First, given $(\Delta,\Phi) \in \mathcal{E}$, we solve the optimal buy/sell mechanisms. Then, we use these mechanisms and the equilibrium equations to determine the new reservation value and distributions $(\hat{\Delta},\hat{\Phi}) \in \mathcal{E}$.

Solving for the optimal bid and ask functions: Consider the pair $(\Delta,\Phi) \in \mathcal{E}$. We want to apply Propositions 11 and 12 to solve for the optimal buying and selling mechanisms under private information.

It is convenient to assume that Φ satisfies equation (33) so it is Lipschitz continuous; however, we know that $\Phi(\theta)$ must be zero in equilibrium if $\Delta(\theta) < 0$. Due to this reason, we work with an adjusted version of Φ when deriving the optimal buying and selling. For each α, define

$$\Delta^{-1}(\alpha,x) = \begin{cases} \inf\{v; \Delta(\alpha,v) \geq x\} & \text{if } \Delta(\alpha,\bar{v}) \geq x \\ \bar{v} & \text{if } \Delta(\alpha,\bar{v}) < x \end{cases}, \quad \text{and} \tag{34}$$

$$\tilde{\Phi}(\alpha,v) = \begin{cases} \Phi(\alpha,v) - \Phi(\alpha,\Delta^{-1}(\alpha,0)) & \text{if } \Delta(\alpha,v) \geq 0 \\ 0 & \text{if } \Delta(\alpha,v) < 0 \end{cases} \tag{35}$$

where $x \in [\underline{\Delta},\bar{\Delta}]$. Equation (31) implies that Δ is strictly increasing in v given α for all $x \in [\Delta(\alpha,\underline{v}),\Delta(\alpha,\bar{v})]$, so its inverse is a bijection in this set. Also, because $\Delta(\alpha,\bar{v}) > \Delta(\alpha,0) \geq 0$ by equation (29), $\tilde{\Phi}(\alpha,\cdot)$ has a non-degenerated support $[\Delta^{-1}(\alpha,0),\bar{v}]$.

Let M_o and M_n be given by

$$M_o(x;\alpha) = \frac{\tilde{\Phi}(\alpha,\Delta^{-1}(\alpha,x))}{\tilde{\Phi}(\alpha,\bar{v})} \quad \text{and} \quad M_n(x;\alpha) = \frac{F(\alpha,\Delta^{-1}(\alpha,x)) - \tilde{\Phi}(\alpha,\Delta^{-1}(\alpha,x))}{F(\alpha,\bar{v}) - \tilde{\Phi}(\alpha,\bar{v})} \tag{36}$$

for $x \in [\Delta(\alpha,\underline{v}),\Delta(\alpha,\bar{v})]$ and given α. The following lemma shows that M_o and M_n as defined above are continuous on $(\Delta,\Phi) \in \mathcal{E}$.

Lemma 10. *Consider any sequence* $\{\Delta_l, \Phi_l\}_l \subset \mathcal{E}$ *converging to a point* $(\Delta^*, \Phi^*) \in \mathcal{E}$ *in the sup norm. Then,* $\{M_{lo}, M_{ln}\}_l$ *also converge to* (M_o^*, M_n^*) *in the sup norm.*

Proof. Consider any sequence $\{\Delta_l, \Phi_l\}_l \subset \mathcal{E}$ converging to $(\Delta^*, \Phi^*) \in \mathcal{E}$.

Let us first show that $\Delta_l^{-1}(\alpha, x)$ converges to $\Delta^{*-1}(\alpha, x)$, as defined in equation (34). First note that equation (34) implies that $\{\Delta_l^{-1}(\alpha, x)\}_l$ is Lipshitz continuous with constant $1/b$, so it has at least one convergent sub-sequence. Moreover, any convergent sub-sequence has to converge to $\Delta^{*-1}(\alpha, x)$. Note that a sequence converge if, and only if, it has at least one converging sub-sequence and all converging sub-sequences converge to the same point. To see that any convergent sub-sequence has to converge to $\Delta^{*-1}(\alpha, x)$, consider a converging sub-sequence and pick first $x \in \left(\Delta^*(\alpha, \underline{v}), \Delta^*(\alpha, \bar{v})\right)$. Note that for l large enough we must also have that $x \in \left(\Delta_l(\alpha, \underline{v}), \Delta_l(\alpha, \bar{v})\right)$. Then, from equation (31), we have that

$$\begin{aligned} \left|\Delta^*\left(\Delta^{*-1}(\alpha, x)\right) - \Delta_l\left(\Delta^{*-1}(\alpha, x)\right)\right| &= \left|\Delta_l\left(\Delta_l^{-1}(\alpha, x)\right) - \Delta_l\left(\Delta^{*-1}(\alpha, x)\right)\right| \geq b\left|\Delta_l^{-1}(\alpha, x) - \Delta^{*-1}(\alpha, x)\right| \\ \implies \quad \left|\Delta_l^{-1}(\alpha, x) - \Delta^{*-1}(\alpha, x)\right| &\leq \frac{1}{b}\left|\Delta^*\left(\Delta^{*-1}(\alpha, x)\right) - \Delta_l\left(\Delta^{*-1}(\alpha, x)\right)\right|. \end{aligned}$$

Note that $\left|\Delta^*\left(\Delta^{*-1}(\alpha, x)\right) - \Delta_l\left(\Delta^{*-1}(\alpha, x)\right)\right|$ goes to zero since Δ_l converges to Δ^* in the sup norm. Therefore, $\Delta_l^{-1}(\alpha, x)$ converge to $\Delta^{*-1}(\alpha, x)$ for $x \in \left(\Delta^*(\alpha, \underline{v}), \Delta^*(\alpha, \bar{v})\right)$.

Now for $x < \Delta^*(\alpha, \underline{v})$, note that for l large enough we must also have that $x < \Delta_(\alpha, \underline{v})$ so $\Delta_l^{-1}(\alpha, x) = \Delta^{*-1}(\alpha, x) = \underline{v}$. If $x > \Delta^*(\alpha, \bar{v})$, then for l large enough we must also have that $x > \Delta_l(\alpha, \bar{v})$ so $\Delta_l^{-1}(\alpha, x) = \Delta^{*-1}(\alpha, x) = \bar{v}$. From the above arguments we can conclude that the convergence occurs everywhere but at the points $x = \Delta^*(\alpha, \underline{v})$ and $x = \Delta^*(\alpha, \bar{v})$. But since the function is Lipshitz continuous, it must also converge at these points at the same rate.

Now let us show that $\tilde{\Phi}_l$, defined by (35), converges to $\tilde{\Phi}^*$ defined by (35). For each α, we have that

$$\tilde{\Phi}_l(\alpha, v) = \begin{cases} \Phi_l(\alpha, v) - \Phi_l(\alpha, \Delta_l^{-1}(\alpha, 0)) & \text{if } \Delta_l(\alpha, v) \geq 0 \\ 0 & \text{if } \Delta_l(\alpha, v) < 0 \end{cases}.$$

We already showed that Δ_l^{-1} converges in the sup norm. The proof that $\tilde{\Phi}_l$ converges to $\tilde{\Phi}^*$ follows similar steps. $\{\tilde{\Phi}_l(\alpha, v)\}_l$ is Lipshitz continuous with constant $\bar{\kappa}_v\{f(\alpha, v)\}$, so it has at least one convergent sub-sequence. For (α, v) such that $\Delta^*(\alpha, v) > 0$,

$$|\tilde{\Phi}_l(\alpha, v) - \tilde{\Phi}^*(\alpha, v)| \leq 2 \sup_\theta |\Phi_l(\theta) - \Phi^*(\theta)| + \bar{\kappa}_v\{f(\alpha, v)\} \sup_\theta |\Delta_l^{-1}(\theta) - \Delta^{-1*}(\theta)|.$$

The above converges because $\tilde{\Phi}_l \to \Phi^*$ and $\Delta_l^{-1} \to \Delta^{*-1}$. For (α, v) such that $\Delta^*(\alpha, v) < 0$, we have that $\tilde{\Phi}_l(\alpha, v) = \tilde{\Phi}^*(\alpha, v) = 0$ for l large enough. And the convergence for (α, v) such that $\Delta^*(\alpha, v) = 0$ can be concluded from the continuity of $\tilde{\Phi}^*(\alpha, v)$.

Finally, let us show that $\{M_{lo}, M_{ln}\}$ converge to (M_o^*, M_n^*). Note that

$$|M_{lo}(x; \alpha) - M_o^*(x; \alpha)| = \left|\frac{\tilde{\Phi}_l(\alpha, \Delta_l^{-1}(\alpha, x))}{\tilde{\Phi}_l(\alpha, \bar{v})} - \frac{\tilde{\Phi}^*(\alpha, \Delta^{*-1}(\alpha, x))}{\tilde{\Phi}^*(\alpha, \bar{v})}\right|$$

$$
\begin{aligned}
&= \left| \frac{\tilde{\Phi}^*(\alpha,\bar{v})\tilde{\Phi}_l(\alpha,\Delta_l^{-1}(\alpha,x)) - \tilde{\Phi}_l(\alpha,\bar{v})\tilde{\Phi}^*(\alpha,\Delta^{*-1}(\alpha,x))}{\tilde{\Phi}_l(\alpha,\bar{v})\tilde{\Phi}^*(\alpha,\bar{v})} \right| \\
&\leq \left| \frac{\tilde{\Phi}^*(\alpha,\bar{v})\tilde{\Phi}_l(\alpha,\Delta_l^{-1}(\alpha,x)) - \tilde{\Phi}^*(\alpha,\bar{v})\tilde{\Phi}_l(\alpha,\Delta^{*-1}(\alpha,x))}{\tilde{\Phi}_l(\alpha,\bar{v})\tilde{\Phi}^*(\alpha,\bar{v})} \right| + \\
&\quad \left| \frac{\tilde{\Phi}^*(\alpha,\bar{v})\tilde{\Phi}_l(\alpha,\Delta^{*-1}(\alpha,x)) - \tilde{\Phi}_l(\alpha,\bar{v})\tilde{\Phi}^*(\alpha,\Delta^{*-1}(\alpha,x))}{\tilde{\Phi}_l(\alpha,\bar{v})\tilde{\Phi}^*(\alpha,\bar{v})} \right| \\
&\leq \left| \frac{\tilde{\Phi}_l(\alpha,\Delta_l^{-1}(\alpha,x)) - \tilde{\Phi}_l(\alpha,\Delta^{*-1}(\alpha,x))}{\tilde{\Phi}_l(\alpha,\bar{v})} \right| + \\
&\quad \left| \frac{\tilde{\Phi}^*(\alpha,\bar{v})\tilde{\Phi}_l(\alpha,\Delta^{*-1}(\alpha,x)) - \tilde{\Phi}^*(\alpha,\bar{v})\tilde{\Phi}^*(\alpha,\Delta^{*-1}(\alpha,x))}{\tilde{\Phi}_l(\alpha,\bar{v})\tilde{\Phi}^*(\alpha,\bar{v})} \right| + \\
&\quad \left| \frac{\tilde{\Phi}^*(\alpha,\bar{v})\tilde{\Phi}^*(\alpha,\Delta^{*-1}(\alpha,x)) - \tilde{\Phi}_l(\alpha,\bar{v})\tilde{\Phi}^*(\alpha,\Delta^{*-1}(\alpha,x))}{\tilde{\Phi}_l(\alpha,\bar{v})\tilde{\Phi}^*(\alpha,\bar{v})} \right| \\
&\leq \frac{\bar{\kappa}\sup_\theta\{f(\theta)\}}{\tilde{\Phi}_l(\alpha,\bar{v})} \times \left| \Delta_l^{-1}(\alpha,x) - \Delta^{*-1}(\alpha,x) \right| + \\
&\quad \frac{1}{\tilde{\Phi}_l(\alpha,\bar{v})} \times \left| \tilde{\Phi}_l(\alpha,\Delta^{*-1}(\alpha,x)) - \tilde{\Phi}^*(\alpha,\Delta^{*-1}(\alpha,x)) \right| + \\
&\quad \frac{\tilde{\Phi}^*(\alpha,\Delta^{*-1}(\alpha,x))}{\tilde{\Phi}_l(\alpha,\bar{v})\tilde{\Phi}^*(\alpha,\bar{v})} \times \left| \tilde{\Phi}^*(\alpha,\bar{v}) - \tilde{\Phi}_l(\alpha,\bar{v}) \right|.
\end{aligned}
$$

In the last equation, since Δ_l^{-1} converges uniformly to Δ^{*-1} and $\tilde{\Phi}_l$ converges uniformly to $\tilde{\Phi}^*$, all the terms multiplying outside the norms converge uniformly to strictly positive numbers evaluated at $\tilde{\Phi}^*$ and the terms inside the norms converge uniformly to zero. Therefore, M_{lo} converges uniformly to M_o^*. The proof that M_{ln} converges uniformly to M_l^* follows the same steps and we ommit them here. □

For each expertise α, $M_o(\cdot;\alpha)$ and $M_n(\cdot;\alpha)$ are probability distributions and satisfy all the properties needed to apply Propositions 11 and 12. That is, they have a non-empty support in an interval and a density that is essentially bounded above and away from zero. We show this below.

Lemma 11. *Consider $(\Delta,\Phi) \in \mathcal{E}$. Then, for each α, $M_o(x;\alpha)$ and $M_n(x;\alpha)$ defined by equation (36) have support $[\max\{\Delta(\alpha,\underline{v}),0\},\Delta(\alpha,\bar{v})]$ and $[\Delta(\alpha,\underline{v}),\Delta(\alpha,\bar{v})]$, and densities $m_o(x;\alpha)$ and $m_n(x;\alpha)$ that are essentially bounded above and away from zero.*

Proof. We will show the results for $M_o(\cdot;\alpha)$. We can use similar arguments to show the results for $M_n(\cdot;\alpha)$, but we omit them here to keep the proof short.

The support of $M_o(\cdot;\alpha)$, given α, is $[\max\{\Delta(\alpha,\underline{v}),0\},\Delta(\alpha,\bar{v})]$ because for $x > \Delta(\alpha,\underline{v})$ the inverse $\Delta^{-1}(\alpha,x) = \bar{v}$ so $M_o(\cdot;\alpha)$ is constant; and for $x < \max\{\Delta(\alpha,\underline{v}),0\}$ either the inverse $\Delta^{-1}(\alpha,x) = \underline{v}$ is constant, or $\tilde{\Phi}(\alpha,\Delta^{-1}(\alpha,x)) = 0$ and is also constant. Similarly, the support of $M_n(\cdot;\alpha)$, given α, is $[\Delta(\alpha,\underline{v}),\Delta(\alpha,\bar{v})]$.

Equation (29) and (31) imply that $\Delta(\alpha, \bar{v}) > 0$ so $M_o(\cdot;\alpha)$ has non-degenerated support. From equation (31), for each α, we know that Δ^{-1} is Lipschitz continuous with constant $\frac{1}{b}$, so it has a derivative almost everywhere which we denote by $d\Delta^{-1}(\alpha, \cdot)$. Again, using equation (31), $d\Delta^{-1}$ is bounded below by $\frac{1}{(1+\kappa)b}$ almost everywhere. So $M_o(x;\alpha) = \int_{\max\{\Delta(\alpha,\underline{v}),0\}}^{x} m_o(u;\alpha)du$ where $m_o(u;\alpha) = \tilde{\phi}(\Delta^{-1}(\alpha, u))d\Delta^{-1}(u)$ for $u \in [\max\{\Delta(\alpha,\underline{v}),0\}, \Delta(\alpha,\underline{v})]$. For given α, $\tilde{\phi}(\alpha,\cdot)$ is bounded below by $\underline{\kappa}\inf_v\{f(\alpha,v)\}$ in its support, and $d\Delta^{-1}$ is essentially bounded below by $\frac{1}{(1+\kappa)b}$ almost everywhere. Then, we can conclude that m_o is essentially bounded below by $\underline{\kappa}\inf_v\{f(\alpha,v)\}\frac{1}{(1+\kappa)b} > 0$. □

Now we apply Propositions 11 and 12 to define the bid and ask prices. The ask price of an owner with reservation value $\Delta_o = \Delta(\theta_o)$ in meeting a non-owner with expertise α_n under private information is

$$ask_o(\Delta_o;\alpha_n) = c_n^{-1}(\Delta_o;\alpha_n), \tag{37}$$

where c_n^{-1} is given in Proposition 11 for given $M_n(\cdot;\alpha_n)$. The bid price of a non-owner with reservation value $\Delta_n = \Delta(\theta_n)$ in meeting an owner with expertise α_o under private information is

$$bid_n(\Delta_n;\alpha_o) = c_o^{-1}(\Delta_n;\alpha_o) \tag{38}$$

where c_o^{-1} is given in Proposition 12 for given $M_n(\cdot;\alpha_n)$.

Lemma 12. *Consider a sequence $\{\Delta_l, \Phi_l\}_l \subset \mathcal{E}$ converging to $(\Delta^*, \Phi^*) \in \mathcal{E}$ in the sup norm. Then, $\{ask_{lo}, bid_{ln}\}_l$ converge to (ask_o^*, bid_n^*) in the L^1 norm. That is, for all α*

$$\lim_l \int_{\underline{\Delta}}^{\bar{\Delta}} |ask_{lo}(x;\alpha) - ask_o^*(x;\alpha)|dx = \lim_l \int_{\underline{\Delta}}^{\bar{\Delta}} |bid_{lo}(x;\alpha) - bid_o^*(x;\alpha)|dx = 0. \tag{39}$$

Proof. We prove the result for ask_o and omit it for bid_n since it is analogous. Let us start fixing an expertise level α.

The ask function $ask_o^*(\cdot;\alpha)$ the unique maximizer of profits almost everywhere (that is, except in a set of measure zero). To see this note that, from the proof of proposition 11, for a given x the profit of the seller is given by

$$\int p(\Delta_n)\left[c_n(\Delta_n;\alpha) - x\right]dM_n(\cdot;\alpha) - \int \left[H_n(M_n(\Delta_n);\alpha) - \bar{H}_n(M_n(\Delta_n);\alpha)\right]dp(\Delta_n).$$

Then, if there is another $ask \neq ask_o^*(x;\alpha)$ that also maximizes profit we must have that

$$\int_{ask_o^*(x;\alpha)}^{\bar{\Delta}} \left[c_n(\Delta_n;\alpha) - x\right]dM_n(\cdot;\alpha) = \int_{ask}^{\bar{\Delta}} \left[c_n(\Delta_n;\alpha) - x\right]dM_n(\cdot;\alpha).$$

Since $ask_o^*(x;\alpha)$ is the infimum value such that $c_n(\Delta_n) \geq x$, we must have that $ask > ask_o^*(x;\alpha)$; otherwise, the right-hand side of the above equation would have to be smaller than the left-hand side. But then,

$$\int_{ask_o^*(x;\alpha)}^{ask} [c_n(\Delta_n;\alpha) - x]\, dM_n(\cdot;\alpha) = 0.$$

But $M_n(\cdot;\alpha)$ has a density bounded away from zero and, by definition, $c_n(\Delta_n;\alpha) \geq x$ for all $\Delta_n \geq ask_o^*(x;\alpha)$. So the above equation implies that $c_n(\Delta_n;\alpha) = x$ for all $\Delta_n \in [ask_o^*(x;\alpha), ask]$. Since $c_n(\cdot;\alpha)$ is weakly increasing and has bounded support, it can have at most countable flat regions. As a result, there are at most countable many points $x \in [\underline{\Delta}, \bar{\Delta}]$ such that $ask_o^*(x;\alpha)$ is not the unique maximizer of profits.

Now let us show that $\{ask_{lo}(x;\alpha)\}_l$ converges to $ask_o^*(x;\alpha)$ for almost every x. Suppose that was not the case for some α and x, such that $ask_o^*(\cdot;\alpha)$ is the unique maximizer of profits. Then, passing to a sub-sequence if necessary, $\{ask_{lo}(x;\alpha)\}_l$ would converge to some $ask \neq ask_o^*(x;\alpha)$. Because $ask_o^*(\cdot;\alpha)$ is the unique maximizer of profits,

$$[1 - M_o^*(ask_o^*(x;\alpha);\alpha)][ask_o^*(\cdot;\alpha) - x] > [1 - M_o^*(ask;\alpha)][ask - x].$$

But since $\{ask_{lo}(x;\alpha)\}_l$ converges to ask, $\{M_{lo}\}_l$ converges to M_o^* in the sup norm, M_o^* and is of bounded variation, the above equation implies that

$$[1 - M_{lo}(ask_o^*(x;\alpha);\alpha)][ask_o^*(\cdot;\alpha) - x] > [1 - M_{ol}(ask_{lo}(x;\alpha);\alpha)][ask_{lo}(x;\alpha) - x].$$

Which is a contradiction since the ask price $ask_{lo}(x;\alpha)$ maximizes profits given the distribution $-M_{ol}$. Therefore, $\{ask_{lo}(x;\alpha)\}_l$ converges to $ask_o^*(x;\alpha)$ for all x such that $ask_o^*(\cdot;\alpha)$ is the unique maximizer of profits. Since $ask_o^*(\cdot;\alpha)$ the unique maximizer for all but countable many points, $\{ask_{lo}(x;\alpha)\}_l$ converges to $ask_o^*(x;\alpha)$ for all but countable many points. That is, almost everywhere.

Let $A \subset [\underline{\Delta}, \bar{\Delta}]$ be the set of points x such that $ask_o^*(\cdot;\alpha)$ is continuous at x and $\{ask_{lo}(x;\alpha)\}_l$ converges to $ask_o^*(x;\alpha)$. Because $c_n(\cdot;\alpha)$ is weakly increasing, $ask_o^*(\cdot;\alpha)$ is also weakly increasing and, therefore, discontinuous at most countable many points. Since we already showed that $\{ask_{lo}(x;\alpha)\}_l$ converges to $ask_o^*(x;\alpha)$ for all but countable many points, we know that the complement of A, defined A^C, has at most countable many points and therefore, has measure zero.

Now let us show that for all α

$$\lim_l \int_{\underline{\Delta}}^{\bar{\Delta}} |ask_{lo}(x;\alpha) - ask_o^*(x;\alpha)| dx = 0.$$

Pick $\epsilon > 0$ and lets us show we can choose l_ϵ such that $\int_{\underline{\Delta}}^{\bar{\Delta}} |ask_{lo}(x;\alpha) - ask_o^*(x;\alpha)| dx < \epsilon$ for all $l \geq l_\epsilon$. Define $\bar{\epsilon} = \frac{\epsilon}{\bar{\Delta} - \underline{\Delta}}$.

Since the interval $[\underline{\Delta}, \bar{\Delta}]$ is separable, so it is the set $A \subset [\underline{\Delta}, \bar{\Delta}]$. Then we can pick a countable set $E \subset A$ which is dense in A. By continuity of $ask_o^*(\cdot;\alpha)$ in A, for each point $x \in E \subset A$ we can find $a_x, b_x \in A$, with $a_x < b_x$, such that $|ask_o^*(x_1;\alpha) - ask_o^*(x_2;\alpha)| < \bar{\epsilon}/5$ for all $x_1, x_2 \in [a_x, b_x] \cap A$.

Define $B_x = [a_x, b_x] \cap A$.

Now, since $A = \cup_{x \in E} B_x$ and the complement of A, A^C, has measure zero, we can pick a finite subset $E' \subset E$ such that

$$\int_A dx - \int_{\cup_{x \in E'} B_x} dx = \int_{\cap_{x \in E'} B_x^C} dx \leq \frac{\bar{\epsilon}}{5},$$

where B_x^C is the coplement of B_x relative to $[\underline{\Delta}, \bar{\Delta}]$. Then we have that

$$\begin{aligned}
\int_{\underline{\Delta}}^{\bar{\Delta}} |ask_{lo}(x;\alpha) - ask_o^*(x;\alpha)| dx &= \int_A |ask_{lo}(x;\alpha) - ask_o^*(x;\alpha)| dx \\
&= \int_{\cup_{x \in E'} B_x} |ask_{lo}(x;\alpha) - ask_o^*(x;\alpha)| dx + \int_{\cap_{x \in E'} B_x^C} |ask_{lo}(x;\alpha) - ask_o^*(x;\alpha)| dx \\
&\leq \int_{\cup_{x \in E'} B_x} |ask_{lo}(x;\alpha) - ask_o^*(x;\alpha)| dx + \int_{\cap_{x \in E'} B_x^C} [\bar{\Delta} - \underline{\Delta}] dx \\
&= \int_{\cup_{x \in E'} B_x} |ask_{lo}(x;\alpha) - ask_o^*(x;\alpha)| dx + \frac{\bar{\epsilon}[\bar{\Delta} - \underline{\Delta}]}{5},
\end{aligned}$$

where the inequalities come from $ask_{lo}(\cdot;\alpha)$ and $ask_o^*(\cdot;\alpha)$ being weakly increasing and satisfying $a_x \leq \tilde{x} \leq b_x$.

Since E' is a finite set, and $\{ask_{lo}(a_x;\alpha)\}_l$ converges to $ask_o^*(a_x;\alpha)$ and $\{ask_{lo}(b_x;\alpha)\}_l$ converges to $ask_o^*(b_x;\alpha)$ for all a_x, b_x associated with $x \in E'$, we can get an l_ϵ large enough such that $|ask_{lo}(a_x;\alpha) - ask_o^*(a_x;\alpha)| < \bar{\epsilon}/5$ and $|ask_{lo}(b_x;\alpha) - ask_o^*(b_x;\alpha)| < \bar{\epsilon}/5$ for all a_x, b_x associated with $x \in E'$. The key for this is that E' is finite, so the l_ϵ does not depend on the particular x, a_x or b_x.

Consider now $\tilde{x} \in B_x$ for $x \in E'$ and $l \geq l_\epsilon$. Note that

$$\begin{aligned}
|ask_{lo}(\tilde{x};\alpha) - ask_o^*(\tilde{x};\alpha)| &\leq |ask_{lo}(b_x;\alpha) - ask_o^*(a_x;\alpha)| + |ask_o^*(b_x;\alpha) - ask_{lo}(a_x;\alpha)| \\
&\leq |ask_{lo}(b_x;\alpha) - ask_o^*(b_x;\alpha)| + |ask_o^*(b_x;\alpha) - ask_o^*(a_x;\alpha)| \\
&+ \ |ask_o^*(b_x;\alpha) - ask_l^*(a_x;\alpha)| + |ask_l^*(a_x;\alpha) - ask_{lo}(a_x;\alpha)| \\
&= \frac{\bar{\epsilon}}{5} + \frac{\bar{\epsilon}}{5} + \frac{\bar{\epsilon}}{5} + \frac{\bar{\epsilon}}{5} = \frac{4\bar{\epsilon}}{5}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\int_{\underline{\Delta}}^{\bar{\Delta}} |ask_{lo}(x;\alpha) - ask_o^*(x;\alpha)| dx &\leq \int_{\cup_{x \in E'} B_x} |ask_{lo}(x;\alpha) - ask_o^*(x;\alpha)| dx + \frac{\bar{\epsilon}[\bar{\Delta} - \underline{\Delta}]}{5} \\
&\leq \int_{\underline{\Delta}}^{\bar{\Delta}} |ask_{lo}(x;\alpha) - ask_o^*(x;\alpha)| dx + \frac{\bar{\epsilon}[\bar{\Delta} - \underline{\Delta}]}{5} \\
&\leq \int_{\underline{\Delta}}^{\bar{\Delta}} \frac{4\bar{\epsilon}}{5} dx + \frac{\bar{\epsilon}[\bar{\Delta} - \underline{\Delta}]}{5} = \bar{\epsilon}[\bar{\Delta} - \underline{\Delta}] = \epsilon
\end{aligned}$$

for all $l \geq l_\epsilon$. Which concludes the proof. □

Reservation value: Now that we have the bid and ask prices for the meetings under private information, we are in position to define the map of reservation values. Given $(\Delta, \Phi) \in \mathcal{E}$, define

$\hat{\Delta}$ as

$$\hat{\Delta}(\theta) = \frac{\nu + \lambda[\Delta(\theta) + (1-s)\pi_o(\theta) - s\pi_n(\theta)]}{\lambda + \mu + \eta + r}, \tag{40}$$

where π_o and π_n are given by (3) and (4) associated with the bid and ask defined above the distribution $\tilde{\Phi}$ as defined in equation (35).

Lemma 13. *Consider $(\Delta, \Phi) \in \mathcal{E}$, then the function $\hat{\Delta}(\alpha, \nu)$ defined in equation (40) is continuous in ν for each α and satisfies equations (29)-(31).*

Proof. For the continuity, note that equation (31) implies that Δ is continuous it suffices to show that $\hat{\Delta}$ satisfies equations (29)-(31).

Let us start with (29), $\hat{\Delta}(\alpha, 0) \geq 0$. The profit of an owner in a meeting, $\pi_o(\theta)$, is bounded below by zero—the worse he can do is decline any offer and ask a price that is never accepted. The profit of a non-owner in a meeting, $\pi_n(\theta)$, is bounded above by $\Delta(\theta) - 0 = \Delta(\theta)$—the best she can do is to buy the asset with probability one at the lowest reservation value in the support of owners, which is at least zero by equation (35). Then we have that,

$$\begin{aligned}\hat{\Delta}(\alpha, 0) &= \frac{\lambda[\Delta(\alpha,0) + (1-s)\pi_o(\alpha,0) - s\pi_n(\alpha,0)]}{\lambda + \mu + \eta + r} \\ &\geq \frac{\lambda[\Delta(\alpha,0) - s\Delta(\alpha,0)]}{\lambda + \mu + \eta + r} = \frac{\lambda(1-s)\Delta(\alpha,0)}{\lambda + \mu + \eta + r} \geq 0,\end{aligned}$$

where the last inequality comes from $\Delta(\alpha, 0) \geq 0$.

Now let us show the two inequalities in equation (30), $\underline{\Delta} \leq \Delta(\alpha, \nu) \leq \bar{\Delta}$. Let us start with $\Delta(\alpha, \nu) \geq \underline{\Delta} := \underline{\nu}/r$. As before, we have that

$$\begin{aligned}\hat{\Delta}(\theta) &= \frac{\nu + \lambda[\Delta(\theta) + (1-s)\pi_o(\theta) - s\pi_n(\theta)]}{\lambda + \mu + \eta + r} \\ &\geq \frac{\nu + \lambda[\Delta(\theta) - s\Delta(\theta)]}{\lambda + \mu + \eta + r} = \frac{\nu + \lambda(1-s)\Delta(\theta)}{\lambda + \mu + \eta + r} \geq \frac{r + \lambda(1-s)}{\lambda + \mu + \eta + r}\underline{\Delta} \geq \underline{\Delta},\end{aligned}$$

where the last inequalities come from $\pi_o(\theta) \geq 0$, $\pi_n(\theta) \leq \Delta(\theta)$, $\nu \geq \underline{\nu} = r\underline{\Delta}$, $\Delta(\theta) \geq \underline{\Delta}$ and $\underline{\Delta} < 0$. Now let us show the second inequality, that is $\hat{\Delta} \leq \bar{\Delta} := \bar{\nu}/r$. We have that

$$\hat{\Delta}(\theta) = \frac{\nu + \lambda[\Delta(\theta) + (1-s)\pi_o(\theta) - s\pi_n(\theta)]}{\lambda + \mu + \eta + r} \leq \frac{\nu + \lambda[\Delta(\theta) + (1-s)(\bar{\Delta} - \Delta(\theta))]}{\lambda + \mu + \eta + r}.$$

The inequality comes from two reasons. First, the highest profit of an owner is achieved if he sells the asset to the highest valuation investor with probability one, which implies that $\pi_o(\theta) \leq \bar{\Delta} - \Delta(\theta)$. Second, the lowest profit a non-owner can make is zero by declining any offer, which implies that $-\pi_n(\theta) \leq 0$. Rearranging the above inequality and using that $\nu \leq \bar{\nu} = r\bar{\Delta}$ and $\Delta(\theta) \leq \bar{\Delta} = \bar{\nu}/r$, we have that

$$\hat{\Delta}(\theta) \leq \frac{\nu + \lambda[\Delta(\theta) + (1-s)(\bar{\Delta} - \Delta(\theta))]}{\lambda + \mu + \eta + r} \leq \frac{r + \lambda}{\lambda + \mu + \eta + r}\bar{\Delta} \leq \bar{\Delta}.$$

Let us now show that $\frac{\hat{\Delta}(\alpha,\hat{\nu})-\hat{\Delta}(\alpha,\nu)}{\hat{\nu}-\nu} \leq (1+\kappa)b$. Without loss of generality, assume that $\hat{\nu} > \nu$. Then, we have that

$$\begin{aligned}\hat{\Delta}(\alpha,\hat{\nu}) - \hat{\Delta}(\alpha,\nu) = & \frac{\hat{\nu} - \nu + \lambda[\Delta(\alpha,\hat{\nu}) - \Delta(\alpha,\nu)]}{\lambda+\mu+\eta+r} \\ & + \lambda \frac{(1-s)[\pi_o(\alpha,\hat{\nu}) - \pi_o(\alpha,\nu)] - s[\pi_n(\alpha,\hat{\nu}) - \pi_n(\alpha,\nu)]}{\lambda+\mu+\eta+r}.\end{aligned}$$

Note that the profit an owner makes must be decreasing in his reservation value. That is because when selling the asset, the owner gives up his reservation value. Moreover, equation (31) implies that $\hat{\Delta}(\alpha,\hat{\nu}) > \hat{\Delta}(\alpha,\nu)$ since $\hat{\Delta}(\alpha,\hat{\nu}) - \hat{\Delta}(\alpha,\nu) \geq \frac{b(\hat{\nu}-\nu)}{2} > 0$. As a result, we have that $\pi_o(\alpha,\hat{\nu}) - \pi_o(\alpha,\nu) \leq 0$. For a similar reason we know that $\pi_n(\alpha,\hat{\nu}) - \pi_n(\alpha,\nu) \geq 0$, which implies that $-[\pi_n(\alpha,\hat{\nu}) - \pi_n(\alpha,\nu)] \leq 0$. Then we have that

$$\hat{\Delta}(\alpha,\hat{\nu}) - \hat{\Delta}(\alpha,\nu) \leq \frac{\hat{\nu} - \nu + \lambda[\Delta(\alpha,\hat{\nu}) - \Delta(\alpha,\nu)]}{\lambda+\mu+\eta+r}.$$

Note now that $(\Delta,\Phi) \in \mathcal{E}$ and, therefore, $\frac{\Delta(\alpha,\hat{\nu})-\Delta(\alpha,\nu)}{\hat{\nu}-\nu} \leq (1+\kappa)b$. As a result, after dividing both sides by $\hat{\nu}-\nu$, we have that

$$\begin{aligned}\frac{\hat{\Delta}(\alpha,\hat{\nu}) - \hat{\Delta}(\alpha,\nu)}{\hat{\nu}-\nu} \leq & \frac{1+\lambda(1+\kappa)b}{\lambda+\mu+\eta+r} = \left[1 + a\left(1 + \frac{1}{1-a}\right)\right] b \\ = & \frac{1-a+a(1-a)+a}{1-a} b = \left(a + \frac{1}{1-a}\right) b \leq (1+\kappa)\, b,\end{aligned}$$

and we conclude that $\frac{\hat{\Delta}(\alpha,\hat{\nu})-\hat{\Delta}(\alpha,\nu)}{\hat{\nu}-\nu} \leq (1+\kappa)b$.

Now we have to show our last inequality, $\frac{\hat{\Delta}(\alpha,\hat{\nu})-\hat{\Delta}(\alpha,\nu)}{\hat{\nu}-\nu} \geq \frac{b}{2}$. Assume again that $\hat{\nu} > \nu$ so $\Delta(\alpha,\hat{\nu}) > \Delta(\alpha,\nu)$. Then,

$$\begin{aligned}\hat{\Delta}(\alpha,\hat{\nu}) - \hat{\Delta}(\alpha,\nu) = & \frac{\hat{\nu} - \nu + \lambda[\Delta(\alpha,\hat{\nu}) - \Delta(\alpha,\nu)]}{\lambda+\mu+\eta+r} \\ & + \lambda \frac{(1-s)[\pi_o(\alpha,\hat{\nu}) - \pi_o(\alpha,\nu)] - s[\pi_n(\alpha,\hat{\nu}) - \pi_n(\alpha,\nu)]}{\lambda+\mu+\eta+r}.\end{aligned}$$

Let us look at the difference in profits. For $\pi_o(\alpha,\hat{\nu}) - \pi_o(\alpha,\nu)$ we have that

$$\begin{aligned}\pi_o(\alpha,\hat{\nu}) - \pi_o(\alpha,\nu) = & -\xi_o\alpha_o \int (\Delta_n - \Delta(\alpha,\nu))\, \mathbb{1}_{\{\Delta(\alpha,\hat{\nu}) > \Delta_n \geq \Delta(\alpha,\nu)\}} d\frac{\tilde{\Phi}_n(\theta_n)}{1-s} \\ & + \xi_o(1-\alpha_o) \underbrace{\int \left\{ (ask_o(\alpha,\hat{\nu}) - \Delta(\alpha,\hat{\nu}))\, \mathbb{1}_{\{\Delta_n \geq ask_o(\alpha,\hat{\nu})\}} - (ask_o(\alpha,\nu) - \Delta(\alpha,\nu))\, \mathbb{1}_{\{\Delta_n \geq ask_o(\alpha,\nu)\}} \right\} d\frac{\tilde{\Phi}_n(\theta_n)}{1-s}}_{\text{difference in gains from trade with owner designing the trade mechanism}} \\ & + \xi_n \underbrace{\int (1-\alpha_n) \left\{ (bid_n - \Delta(\alpha,\hat{\nu}))\, \mathbb{1}_{\{bid_n \geq \Delta(\alpha,\hat{\nu})\}} - (bid_n - \Delta(\alpha,\nu))\, \mathbb{1}_{\{bid_n \geq \Delta(\alpha,\nu)\}} \right\} d\frac{\tilde{\Phi}_n(\theta_n)}{1-s}}_{\text{difference in gains from trade with non-owner designing the trade mechanism}}.\end{aligned}$$

Note that whether the owner is designing the trade mechanism, or the non-owner is designing it, the mechanism is incentive compatible and individually rational for the owner. As a result, it satisfy the conditions of Lemma 7 and we have that

$$\begin{aligned}\pi_o(\alpha,\hat{\nu})-\pi_o(\alpha,\nu)=&-\xi_o\alpha_o\int(\Delta_n-\Delta(\alpha,\nu))\,\mathbb{1}_{\{\Delta(\alpha,\hat{\nu})>\Delta_n\geq\Delta(\alpha,\nu)\}}d\frac{\tilde{\Phi}_n(\theta_n)}{1-s}\\&-\xi_o(1-\alpha_o)\int_{\Delta(\alpha,\nu)}^{\Delta(\alpha,\hat{\nu})}\left[\int\mathbb{1}_{\{c_n(\Delta_n;\alpha_n)\geq\tilde{\Delta}\}}d\frac{\tilde{\Phi}_n(\theta_n)}{1-s}\right]d\tilde{\Delta}\\&-\xi_n\int_{\Delta(\alpha,\nu)}^{\Delta(\alpha,\hat{\nu})}\left[\int(1-\alpha_n)\mathbb{1}_{\{\Delta_n\geq c_o(\tilde{\Delta};\alpha)\}}d\frac{\tilde{\Phi}_n(\theta_n)}{1-s}\right]d\tilde{\Delta}.\end{aligned}$$

Now we can easily see that

$$\begin{aligned}\pi_o(\alpha,\hat{\nu})-\pi_o(\alpha,\nu)&\geq-[\xi_o\alpha_o+\xi_o(1-\alpha_o)+\xi_n(1-\alpha_n)][\Delta(\alpha,\hat{\nu})-\Delta(\alpha,\nu)]\\&=-[\Delta(\alpha,\hat{\nu})-\Delta(\alpha,\nu)].\end{aligned}$$

We can use very similar arguments to show that the difference in profits for the non-owners, $\pi_n(\alpha,\hat{\nu})-\pi_n(\alpha,\nu)$, satisfy

$$\pi_n(\alpha,\hat{\nu})-\pi_n(\alpha,\nu)\leq\Delta(\alpha,\hat{\nu})-\Delta(\alpha,\nu).$$

Then we have that

$$\begin{aligned}\hat{\Delta}(\alpha,\hat{\nu})-\hat{\Delta}(\alpha,\nu)=&\frac{\hat{\nu}-\nu+\lambda[\Delta(\alpha,\hat{\nu})-\Delta(\alpha,\nu)]}{\lambda+\mu+\eta+r}+\lambda\frac{(1-s)[\pi_o(\alpha,\hat{\nu})-\pi_o(\alpha,\nu)]-s[\pi_n(\alpha,\hat{\nu})-\pi_n(\alpha,\nu)]}{\lambda+\mu+\eta+r}\\\geq&\frac{\hat{\nu}-\nu+\lambda[\Delta(\alpha,\hat{\nu})-\Delta(\alpha,\nu)]}{\lambda+\mu+\eta+r}-\lambda\frac{(1-s)[\Delta(\alpha,\hat{\nu})-\Delta(\alpha,\nu)]+s[\Delta(\alpha,\hat{\nu})-\Delta(\alpha,\nu)]}{\lambda+\mu+\eta+r}=\frac{\hat{\nu}-\nu}{\lambda+\mu+\eta+r}.\end{aligned}$$

Therefore, $\frac{\hat{\Delta}(\alpha,\hat{\nu})-\hat{\Delta}(\alpha,\nu)}{\hat{\nu}-\nu}\geq\frac{1}{\lambda+\mu+\eta+r}=b$, which concludes the proof. □

The previous lemma shows that the map above takes $(\Delta,\Phi)\in\mathcal{E}$ into $\hat{\Delta}$ satisfying the conditions of the set $\mathcal{E}$. Now we show this map is also continuous in the sup norm.

Lemma 14. *Consider any sequence $\{\Delta_l,\Phi_l\}_l\subset\mathcal{E}$ converging to a point $(\Delta^*,\Phi^*)\in\mathcal{E}$ in the sup norm. Then, $\{\hat{\Delta}_l\}_l$ also converge to $\hat{\Delta}^*$ in the sup norm, where $\{\hat{\Delta}_l\}_l$ and $\hat{\Delta}^*$ are defined as in equation (40) based on $\{\Delta_l,\Phi_l\}_l$ and (Δ^*,Φ^*).*

Proof. By equation (40) we have that

$$\hat{\Delta}_l(\theta)=\frac{\nu+\lambda\Delta_l(\theta)}{\lambda+\mu+\eta+r}+\lambda\frac{(1-s_l)\pi_{lo}(\theta)-s_l\pi_{ln}(\theta)}{\lambda+\mu+\eta+r}.$$

The first term in the right-hand side converges in the sup norm because it is just a positive constant multiplying Δ_l, which converges in the sup norm to Δ^*. The second term has a constant, $\lambda/\lambda+\mu+\eta+r$, multiplying $(1-s_l)\pi_{lo}(\theta)$ and $s_l\pi_{ln}(\theta)$. So we just have to show that $(1-s_l)\pi_{lo}(\theta)$ and $s_l\pi_{ln}(\theta)$ converge in the sup norm. We show the result for $(1-s_l)\pi_{lo}(\theta)$, the result for $s_l\pi_{ln}(\theta)$ is analogous.

We have that

$$(1-s_l)\pi_{lo}(\alpha,\nu)=\xi_o\alpha\int(\Delta_l(\theta_n)-\Delta_l(\alpha,\nu))\,\mathbb{1}_{\{\Delta_l(\theta_n)\geq\Delta_l(\alpha,\nu)\}}d\tilde{\Phi}_{ln}(\theta_n)$$

$$+ \zeta_o (1-\alpha) \int \left[ask_{lo}(\Delta_l(\alpha,\nu);\alpha_n) - \Delta_l(\alpha,\nu)\right] \mathbb{1}_{\{\Delta_{ln} \geq ask_{lo}(\Delta_l(\alpha,\nu);\alpha_n)\}} d\tilde{\Phi}_{ln}(\theta_n)$$
$$+ \zeta_n \int (1-\alpha_n) \left[bid_{ln}(\Delta_l(\theta_n);\alpha) - \Delta_l(\alpha,\nu)\right] \mathbb{1}_{\{bid_{ln}(\Delta_l(\theta_n);\alpha) \geq \Delta_l(\alpha,\nu)\}} d\tilde{\Phi}_{ln}(\theta_n).$$

It is easy to see that the first and last term in the right-hand side converge in the sup since Δ_l and Φ_{ln} converge in the sup norm, and $bid_{ln}(\cdot;\alpha)$ converge in L^1.

The middle term in the right-hand side is a bit trickier. To see this note that

$$\int \left[ask_{lo}(\Delta_l(\alpha,\nu);\alpha_n) - \Delta_l(\alpha,\nu)\right] \mathbb{1}_{\{\Delta_{ln} \geq ask_{lo}(\Delta_l(\alpha,\nu);\alpha_n)\}} d\tilde{\Phi}_{ln}(\theta_n)$$
$$= \sum_{\alpha_n} \left[ask_{lo}(\Delta_l(\alpha,\nu);\alpha_n) - \Delta_l(\alpha,\nu)\right] \left[\tilde{\Phi}_{ln}(\alpha_n,\bar{\nu}) - \tilde{\Phi}_{ln}\left(\alpha_n, \Delta_l^{-1}(\alpha_n, ask_{lo}(\Delta_l(\alpha,\nu);\alpha_n))\right)\right].$$

Though $ask_{lo}(\cdot;\alpha_n)$ converges in the L^1 norm, it doesn't have to converge pointwise. In principle, this could make the above term diverge in the same points that $ask_{lo}(\cdot;\alpha_n)$ diverge. The reason the term above converges is that, for each α_n,

$$\left[ask_{lo}(\Delta_l(\alpha,\nu);\alpha_n) - \Delta_l(\alpha,\nu)\right] \left[\tilde{\Phi}_{ln}(\alpha_n,\bar{\nu}) - \tilde{\Phi}_{ln}\left(\alpha_n, \Delta_l^{-1}(\alpha_n, ask_{lo}(\Delta_l(\alpha,\nu);\alpha_n))\right)\right]$$

is the expected profit an owner of type (α,ν) obtain in a meeting with a non-owner under private information. As we argue in the proof of lemma 12, these profits converge in the sup norm. That is because the points in which $ask_{lo}(\cdot;\alpha_n)$ could diverge are points in which there are more than one ask price which maximizes profits and the profits itself converge. As we show in the lemma, if that was not the case the profits in the limit could be improved by picking an ask price used in the sequence.

We can now conclude that $(1-s_l)\pi_{lo}$ converges in the sup norm to $(1-s^*)\pi_o^*$. The proof that $s_l\pi_{ln}$ converges in the sup norm to $s^*\pi_n^*$ is analogous and we omit it here. As a result, $\{\hat{\Delta}_l\}_l$ also converge to $\hat{\Delta}^*$ in the sup norm, which concludes the proof. □

Distribution: After defining the map of reservation values, we turn our attention to the distributions. Given $(\Delta,\Phi) \in \mathcal{E}$, define the distribution $\hat{\Phi}$ as

$$\hat{\Phi}(\theta) = \int_{\tilde{\theta} \leq \theta} \hat{\phi}(\tilde{\theta}) d\tilde{\theta}, \tag{41}$$

where

$$\hat{\phi}(\theta) = \begin{cases} \frac{[\lambda \bar{q}_n(\theta)+\eta] f(\theta)}{\lambda[\bar{q}_o(\theta)+\bar{q}_n(\theta)]+\mu+\eta} & \text{if } \Delta(\theta) \geq 0 \\ \frac{\eta f(\theta)}{\lambda+\mu+\eta} & \text{if } \Delta(\theta) < 0 \end{cases}, \tag{42}$$

and we obtain $\bar{q}_n(\theta)$ and $\bar{q}_o(\theta)$ from equations (9)-(11), and the distribution $\tilde{\Phi}$ as defined in equation (35).

Similarly to the result for $\hat{\Delta}$ given in Lemma 13, the construction of $\hat{\Phi}$ implies that the constraints we impose in the set $\mathcal{E}$ are satisfied.

Lemma 15. *Consider $(\Delta,\Phi)\in\mathcal{E}$, then the function $\hat{\Phi}(\alpha,\nu)$ defined in equation (41) is continuous in ν for each α and satisfies equations (32)-(33).*

Proof. First note that $\hat{\Phi}\in\mathcal{C}^0(\Theta_M)$ since it is the integral of the function $\hat{\phi}$ which is bounded. So we just need to show that

$$0\leq\Phi(\alpha,\nu)\leq\bar{\kappa}F(\alpha,\nu)\ \text{ and }\ \underline{\kappa}\frac{F(\alpha,\hat{\nu})-F(\alpha,\nu)}{\hat{\nu}-\nu}\leq\frac{\hat{\Phi}(\alpha,\hat{\nu})-\hat{\Phi}(\alpha,\nu)}{\hat{\nu}-\nu}\leq\bar{\kappa}\frac{F(\alpha,\hat{\nu})-F(\alpha,\nu)}{\hat{\nu}-\nu}$$

for all $(\alpha,\nu),(\alpha,\hat{\nu})\in\Theta_M$.

The above inequalities come from the fact that $\hat{\Phi}=\int\hat{\phi}$ and by noticing that $\underline{\kappa}f\leq\hat{\phi}\leq\bar{\kappa}f$. It is easy to see how these imply that $\hat{\Phi}$ is non-negative, bounded by $\bar{\kappa}F$, and that changes in $\hat{\Phi}$ are bounded by changes in F. For the last two inequalities, without loss of generality consider $\hat{\nu}>\nu$ and any α. Then,

$$\hat{\Phi}(\alpha,\hat{\nu})-\hat{\Phi}(\alpha,\nu)=\int_{\nu}^{\hat{\nu}}\hat{\phi}(\alpha,\tilde{\nu})d\tilde{\nu}\leq\int_{\nu}^{\hat{\nu}}\bar{\kappa}f(\alpha,\tilde{\nu})d\tilde{\nu}=\bar{\kappa}[F(\alpha,\hat{\nu})-F(\alpha,\nu)],$$

which implies that $\frac{\hat{\Phi}(\alpha,\hat{\nu})-\hat{\Phi}(\alpha,\nu)}{\hat{\nu}-\nu}\leq\bar{\kappa}\frac{F(\alpha,\hat{\nu})-F(\alpha,\nu)}{\hat{\nu}-\nu}$. In a similar way,

$$\hat{\Phi}(\alpha,\hat{\nu})-\hat{\Phi}(\alpha,\nu)=\int_{\nu}^{\hat{\nu}}\hat{\phi}(\alpha,\tilde{\nu})d\tilde{\nu}\geq\int_{\nu}^{\hat{\nu}}\underline{\kappa}f(\alpha,\tilde{\nu})d\tilde{\nu}=\underline{\kappa}[F(\alpha,\hat{\nu})-F(\alpha,\nu)],$$

which implies that $\frac{\hat{\Phi}(\alpha,\hat{\nu})-\hat{\Phi}(\alpha,\nu)}{\hat{\nu}-\nu}\geq\underline{\kappa}\frac{F(\alpha,\hat{\nu})-F(\alpha,\nu)}{\hat{\nu}-\nu}$. Which concludes the proof. □

The previous lemma shows that the map above takes $(\Delta,\Phi)\in\mathcal{E}$ into a $\hat{\Phi}$ satisfying the conditions of the set $\mathcal{E}$. Now we show this map is also continuous in the sup norm.

Lemma 16. *Consider any sequence $\{\Delta_l,\Phi_l\}_l\subset\mathcal{E}$ converging to a point $(\Delta^*,\Phi^*)\in\mathcal{E}$ in the sup norm. Then, $\{\hat{\Phi}_l\}_l$ also converge to $\hat{\Phi}^*$ in the sup norm, where $\{\hat{\Phi}_l\}_l$ and $\hat{\Phi}^*$ are defined as in equation (41) based on $\{\Delta_l,\Phi_l\}_l$ and (Δ^*,Φ^*).*

Proof. By definition we have that

$$\begin{aligned}|\hat{\Phi}_l(\theta)-\hat{\Phi}^*(\theta)|&=\int_{\tilde{\theta}\leq\theta}|\hat{\phi}_l(\tilde{\theta})-\hat{\phi}^*(\tilde{\theta})|d\tilde{\theta}\\&\leq\int_{\tilde{\theta}\leq\theta}[\bar{\kappa}-\underline{\kappa}]f(\tilde{\theta})\mathbb{1}_{\{\min\{\Delta_l(\tilde{\theta}),\Delta^*(\tilde{\theta})\}<0<\max\{\Delta_l(\tilde{\theta}),\Delta^*(\tilde{\theta})\}\}}d\tilde{\theta}\\&+\int_{\tilde{\theta}\leq\theta}\left|\frac{[\lambda\bar{q}_{ln}(\tilde{\theta})+\eta]f(\tilde{\theta})}{\lambda[\bar{q}_{lo}(\tilde{\theta})+\bar{q}_{ln}(\tilde{\theta})]+\mu+\eta}-\frac{[\lambda\bar{q}^*_n(\tilde{\theta})+\eta]f(\tilde{\theta})}{\lambda[\bar{q}_o(\tilde{\theta})+\bar{q}^*_n(\tilde{\theta})]+\mu+\eta}\right|\mathbb{1}_{\{\min\{\Delta_l(\tilde{\theta}),\Delta^*(\tilde{\theta})\}\geq0\}}d\tilde{\theta}.\end{aligned}$$

To understand this inequality, note that, when $\Delta_l(\tilde{\theta})$ and $\Delta^*(\tilde{\theta})$ are negative, $\hat{\phi}_l(\tilde{\theta})=\hat{\phi}^*(\tilde{\theta})=\eta f(\theta)/\lambda+\mu+\eta$; when one of $\Delta_l(\tilde{\theta})$, $\Delta^*(\tilde{\theta})$ is positive and the other is negative, the difference is bounded above by $[\bar{\kappa}-\underline{\kappa}]f(\tilde{\theta})$; and when both are positive the difference is $|\hat{\phi}_l(\theta)-\hat{\phi}^*(\theta)|$, where $\hat{\phi}_l(\theta)$ and $\hat{\phi}^*(\theta)$ are given by the first item in equation (42).

For the first term after the inequality, we have that

$$\int_{\tilde{\theta}\leq\theta}[\bar{\kappa}-\underline{\kappa}]f(\tilde{\theta})\mathbb{1}_{\{\min\{\Delta_l(\tilde{\theta}),\Delta^*(\tilde{\theta})\}<0<\max\{\Delta_l(\tilde{\theta}),\Delta^*(\tilde{\theta})\}\}}d\tilde{\theta}$$

$$\leq \sup_{\tilde{\theta}}\{[\bar{\kappa}-\underline{\kappa}]f(\tilde{\theta})\}\sum_{\alpha}\sup_{\tilde{\theta}}\{\Delta_l^{-1}(\alpha,0)-\Delta^{*-1}(\alpha,0)\}.$$

As we've shown in the proof of Lemma 10, $\Delta_l^{-1}(\alpha,0)$ converges to $\Delta^{*-1}(\alpha,0)$. Therefore, the above term converges to zero.

For the last term after the inequality, first note that

$$\int_{\tilde{\theta}\leq\theta}\left|\frac{[\lambda\bar{q}_{ln}(\tilde{\theta})+\eta]f(\tilde{\theta})}{\lambda[\bar{q}_{lo}(\tilde{\theta})+\bar{q}_{ln}(\tilde{\theta})]+\mu+\eta}-\frac{[\lambda\bar{q}_n^*(\tilde{\theta})+\eta]f(\tilde{\theta})}{\lambda[\bar{q}_o(\theta)+\bar{q}_n^*(\tilde{\theta})]+\mu+\eta}\right|\mathbb{1}_{\{\min\{\Delta_l(\tilde{\theta}),\Delta^*(\tilde{\theta})\}\geq 0\}}d\tilde{\theta}$$
$$\leq \int_{\tilde{\theta}\leq\theta}\left|\frac{[\lambda\bar{q}_{ln}(\tilde{\theta})+\eta]f(\tilde{\theta})}{\lambda[\bar{q}_{lo}(\tilde{\theta})+\bar{q}_{ln}(\tilde{\theta})]+\mu+\eta}-\frac{[\lambda\bar{q}_n^*(\tilde{\theta})+\eta]f(\tilde{\theta})}{\lambda[\bar{q}_o(\tilde{\theta})+\bar{q}_n^*(\tilde{\theta})]+\mu+\eta}\right|d\tilde{\theta}.$$

Now, note that

$$\left|\frac{d}{dq_o}\left(\frac{[\lambda\bar{q}_n+\eta]f(\tilde{\theta})}{\lambda[\bar{q}_o+\bar{q}_n]+\mu+\eta}\right)\right|=\frac{\lambda[\lambda\bar{q}_n+\eta]f(\tilde{\theta})}{(\lambda[\bar{q}_o+\bar{q}_n]+\mu+\eta)^2}\leq\frac{\lambda(\lambda+\eta)\sup_{\tilde{\theta}}f(\tilde{\theta})}{(\mu+\eta)^2},$$

and

$$\left|\frac{d}{dq_n}\left(\frac{[\lambda\bar{q}_n+\eta]f(\tilde{\theta})}{\lambda[\bar{q}_o+\bar{q}_n]+\mu+\eta}\right)\right|=\frac{\lambda[\lambda(\bar{q}_{lo}+\bar{q}_n)+\mu+\eta]-\lambda[\lambda\bar{q}_n+\eta]}{[\lambda(\bar{q}_{lo}+\bar{q}_n)+\mu+\eta]^2}f(\tilde{\theta})$$
$$=\frac{\lambda[\lambda\bar{q}_{lo}+\mu]}{[\lambda(\bar{q}_{lo}+\bar{q}_n)+\mu+\eta]^2}f(\tilde{\theta})\leq\frac{\lambda(\lambda+\mu)\sup_{\tilde{\theta}}f(\tilde{\theta})}{(\mu+\eta)^2}.$$

So we can write that

$$\int_{\tilde{\theta}\leq\theta}\left|\frac{[\lambda\bar{q}_{ln}(\tilde{\theta})+\eta]f(\tilde{\theta})}{\lambda[\bar{q}_{lo}(\tilde{\theta})+\bar{q}_{ln}(\tilde{\theta})]+\mu+\eta}-\frac{[\lambda\bar{q}_n^*(\tilde{\theta})+\eta]f(\tilde{\theta})}{\lambda[\bar{q}_o(\theta)+\bar{q}_n^*(\tilde{\theta})]+\mu+\eta}\right|\mathbb{1}_{\{\min\{\Delta_l(\tilde{\theta}),\Delta^*(\tilde{\theta})\}\geq 0\}}d\tilde{\theta}$$
$$\leq\frac{\lambda(\lambda+\mu+\eta)\sup_{\tilde{\theta}}f(\tilde{\theta})}{(\mu+\eta)^2}\int_{\tilde{\theta}\leq\theta}\left|\bar{q}_{ln}(\tilde{\theta})-\bar{q}_n^*(\tilde{\theta})\right|+\left|\bar{q}_{lo}(\tilde{\theta})-\bar{q}_o^*(\tilde{\theta})\right|d\tilde{\theta}$$
$$\leq\frac{\lambda(\lambda+\mu+\eta)\sup_{\tilde{\theta}}f(\tilde{\theta})}{(\mu+\eta)^2}\int_{\tilde{\theta}}\int_{\theta_o}\left|q_l(\theta_o,\tilde{\theta})-q^*(\theta_o,\tilde{\theta})\right|+\int_{\theta_n}\left|q_l(\tilde{\theta},\theta_n)-q^*(\tilde{\theta},\theta_n)\right|d\theta.$$

It then suffices to show that $\bar{q}_{lo}(\alpha,\nu)$ and $\bar{q}_{ln}(\alpha,\nu)$ converge in the L^1 norm to $\bar{q}_o^*(\alpha,\nu)$ and $\bar{q}_n^*(\alpha,\nu)$. By definition,

$$q(\theta_o,\theta_n)=\mathbb{1}_{\{\Delta_n\geq\Delta_o\}}-\xi_o(1-\alpha_o)\mathbb{1}_{\{ask_o>\Delta_n\geq\Delta_o\}}-\xi_n(1-\alpha_n)\mathbb{1}_{\{\Delta_n\geq\Delta_o>bid_n\}}.$$

But since Δ_l converges in the sup norm to Δ^*, and ask_l and bid_l converge in the L^1 norm to ask^* and bid^*, the measure of points (θ_o,θ_n) such that either

$$|\mathbb{1}_{\{\Delta_{ln}\geq\Delta_{lo}\}}-\mathbb{1}_{\{\Delta_n^*\geq\Delta_o^*\}}|=1 \text{ or } |\mathbb{1}_{\{ask_{lo}>\Delta_{ln}\geq\Delta_{lo}\}}-\mathbb{1}_{\{ask_o^*>\Delta_n^*\geq\Delta_o^*\}}|=1$$
$$\text{or } |\mathbb{1}_{\{\Delta_{ln}\geq\Delta_{lo}>bid_{ln}\}}-\mathbb{1}_{\{\Delta_n^*\geq\Delta_o^*>bid_n^*\}}|=1$$

must be converging to zero. This convergence is independent of θ so

$$\int_{\tilde{\theta}\leq\theta}\left|\frac{[\lambda\bar{q}_{ln}(\tilde{\theta})+\eta]f(\tilde{\theta})}{\lambda[\bar{q}_{lo}(\tilde{\theta})+\bar{q}_{ln}(\tilde{\theta})]+\mu+\eta}-\frac{[\lambda\bar{q}_n^*(\tilde{\theta})+\eta]f(\tilde{\theta})}{\lambda[\bar{q}_o(\theta)+\bar{q}_n^*(\tilde{\theta})]+\mu+\eta}\right|\mathbb{1}_{\{\min\{\Delta_l(\tilde{\theta}),\Delta^*(\tilde{\theta})\}\geq 0\}}d\tilde{\theta}$$

goes to zero uniformly and therefore $\hat{\Phi}_l(\theta)$ converges to $\hat{\Phi}^*(\theta)$ in the sup norm. Which concludes the proof. □

Operator and fixed point: Define the map $T : \mathcal{E} \to \mathcal{E}$ associated with equations (40) and (41). Lemmas 13 and 15 imply that $T(\Delta, \Phi) \in \mathcal{E}$ is a well defined map. Moreover, lemmas 16 and 14 imply that T is continuous in the sup norm. Therefore, T has a fixed point.

Lemma 17. *The map T defined above has a fixed point. That is, there exists $(\Delta^*, \Phi^*) \in \mathcal{E}$ such that $T(\Delta^*, \Phi^*) = (\Delta^*, \Phi^*)$.*

Proof. This is a direct application of the Schauder Fixed-Point Theorem. The set $\mathcal{E}$ is convex and compact, and the map T is continuous, so a fixed point exists. □

Unbounded fixed point: So far we have being working with a bounded support for ν given by $\underline{\nu}$ and $\bar{\nu}$. Using these bounds, we showed that we can find a fixed point of $(\Delta^*, \Phi^*) \in \mathcal{E}$ of the operator T. That is, a pair $(\Delta^*, \Phi^*) \in \mathcal{E}$ satisfying equations (40) and (41). Now we show that we can get a fixed point associated with equations (40) and (41) when we take the limit with $\underline{\nu}$ and $\bar{\nu}$ going to minus and plus infinity.

Before taking the limit with $\underline{\nu}$ and $\bar{\nu}$ going to minus and plus infinity, we have to show one result. Let $\iota = \frac{2}{r+\mu+\eta}$ and $\nu_H > 0$ be such that

$$\sum_{\tilde{\alpha}} \int_{\nu}^{\infty} (\tilde{\nu} - \nu) F(\tilde{\alpha}, d\tilde{\nu}) \leq \frac{r+\mu+\eta}{2\lambda} \nu \tag{43}$$

for all $\nu \geq \nu_H$. Note that $\int \nu^2 F(\alpha, d\nu) < \infty$ for all α implies that ν_H above exists.

Lemma 18. *Consider a truncation $[\underline{\nu}, \bar{\nu}]$ with $\underline{\nu} \leq -\lambda\bar{\nu} < \nu_H < \bar{\nu}$, and some $(\Delta, \Phi) \in \mathcal{E}$. If $\Delta(\alpha, \nu) > \iota\nu$ for some α and $\nu \geq \nu_H$, then $(\Delta, \Phi) \in \mathcal{E}$ is not a fixed point of the operator T.*

Proof. Let $\theta = (\alpha, \nu) \in \arg\max_{\tilde{\alpha},\ \tilde{\nu} \in [\nu_H, \bar{\nu}]} \{\Delta(\tilde{\alpha}, \tilde{\nu}) - \iota\tilde{\nu}\}$ and $C = \Delta(\alpha, \nu) - \iota\nu$. Since Δ is continuous in ν, the set of α's is finite, and $[\nu_H, \bar{\nu}]$ is compact, the argmax above exists, (α, ν) is well defined. Moreover, because $\Delta(\tilde{\alpha}, \tilde{\nu}) > \iota\nu$ for some $\tilde{\alpha}$ and $\tilde{\nu} \geq \nu_H$, $C > 0$.

From equation (40), we have that

$$\hat{\Delta}(\theta) = \frac{\nu + \lambda[\Delta(\theta) + (1-s)\pi_o(\theta) - s\pi_n(\theta)]}{\lambda + \mu + \eta + r} \leq \frac{\nu + \lambda[\iota\nu + C + (1-s)\pi_o(\theta)]}{\lambda + \mu + \eta + r}.$$

Now note that

$$(1-s)\pi_o(\theta) \leq \sum_{\tilde{\alpha}} \int_{\tilde{\nu}} [\Delta(\tilde{\alpha}, \tilde{\nu}) - \Delta(\theta)] \mathbb{1}_{\{\Delta(\tilde{\alpha}, \tilde{\nu}) \geq \Delta(\theta)\}} F(\tilde{\alpha}, d\tilde{\nu}).$$

For $\tilde{\nu} < \nu$, we have that $\Delta(\tilde{\alpha}, \tilde{\nu}) < \Delta(\tilde{\alpha}, \nu) \leq \Delta(\alpha, \nu)$ by the definition of (α, ν). Therefore,

$$\begin{aligned}(1-s)\pi_o(\theta) &\leq \sum_{\tilde{\alpha}} \int_{\nu} |\Delta(\tilde{\alpha}, \tilde{\nu}) - \Delta(\theta)| F(\tilde{\alpha}, d\tilde{\nu}) \leq \sum_{\tilde{\alpha}} \int_{\nu} [\iota\tilde{\nu} + C - \Delta(\theta)] F(\tilde{\alpha}, d\tilde{\nu}) \\ &= \iota \sum_{\tilde{\alpha}} \int_{\nu} (\tilde{\nu} - \nu) F(\tilde{\alpha}, d\tilde{\nu}) \leq \iota \frac{r+\mu+\eta}{2\lambda} \nu.\end{aligned}$$

As a result,

$$\begin{aligned}\hat{\Delta}(\theta) \leq & \frac{\nu + \lambda[\iota\nu + C + (1-s)\pi_o(\theta)]}{\lambda+\mu+\eta+r} \leq \frac{\nu + \lambda\left[\iota\nu + C + \iota\frac{r+\mu+\eta}{2\lambda}\nu\right]}{\lambda+\mu+\eta+r} \\ \leq & \frac{2\nu + \frac{2\lambda\nu}{r+\mu+\eta}}{\lambda+\mu+\eta+r} + \frac{\lambda}{\lambda+\mu+\eta+r}C = \frac{2\nu}{r+\mu+\eta} + \frac{\lambda}{\lambda+\mu+\eta+r}C \\ = & \iota\nu + \frac{\lambda}{\lambda+\mu+\eta+r}C < \iota\nu + C = \Delta(\theta).\end{aligned}$$

Since $\hat{\Delta}(\theta) < \Delta(\theta)$, we have that $T(\Delta, \Phi) \neq (\Delta, \Phi)$. This concludes the proof. □

The idea of our proof is simple: we pick a sequence of supports $\{[\underline{\nu}_N, \bar{\nu}_N]\}_N$ and fixed point $\{(\Delta_N, \Phi_N)\}_N$ associated with each of the supports. Then we argue that this sequence has a convergent sub-sequence so the limit must satisfy the same fixed point equations. However, this argument is not that simple because, with unbounded support, we cannot apply the Arzelà-Ascoli theorem so the space is not compact. Therefore, a converging sub-sequence may not exit.

A sub-sequence that converges in the sup norm may not exist, but we can still build a sub-sequence that converges in the compact norm.

Definition 2. *A sequence $\{(\Delta_N, \Phi_N)\}_N$ converges compactly to $\{(\Delta^*, \Phi^*)$ if it converges in the sup norm in every compact sub-set of Θ.*

Let the set $\mathcal{E}_N$ be defined as before, by equations (29)-(33), where the bounds in ν's are $\nu \in [\underline{\nu}_N, \bar{\nu}_N] = [-\lambda N\bar{\nu}, N\bar{\nu}]$ and $\bar{\nu} > \nu_H$ given by equation (43).

Lemma 19. *Consider a sequence $\{(\Delta_N, \Phi_N)\}_N$ of fixed points of T_N defined as before on the intervals $\{[\underline{\nu}_N, \bar{\nu}_N]\}_N$. Then, $\{(\Delta_N, \Phi_N)\}_N$ has a sub-sequence that converges compactly.*

Proof. For this proof we use a diagonal argument. For given $l \leq N$, let $\{(\Delta^l_N, \Phi^l_N)\}_N$ be the sequence $\{(\Delta_N, \Phi_N)\}_{N\geq l}$ but with each $(\Delta_N, \Phi_N) \in \mathcal{E}_N$ truncated in the set $\mathcal{E}_l$. That is, $\Delta^l_N(\alpha, \nu) = \Delta_N(\alpha, \nu)$ for all α and $\nu \in [\underline{\nu}_l, \bar{\nu}_l] \subset [\underline{\nu}_N, \bar{\nu}_N]$.

We build our sub-sequence by induction. Fix $l = 1$. Note that the sequence $\{(\Delta^l_N, \Phi^l_N)\}_N$ satisfy equations (29) and (31)-(33) associated with the bound $[\underline{\nu}_l, \bar{\nu}_l]$. From the previous lemma, we also know $\Delta^l_N(\alpha, \tilde{\nu}) \leq \iota\bar{\nu}_l$, and we can also see that

$$\begin{aligned}\Delta^l_N(\theta) = & \frac{\nu + \lambda[\Delta^l_N(\theta) + (1-s)\pi_o(\theta) - s\pi_n(\theta)]}{\lambda+\mu+\eta+r} \\ \geq & \frac{\nu + \lambda[\Delta^l_N(\theta) - s\pi_n(\theta)]}{\lambda+\mu+\eta+r} \geq \frac{\nu + \lambda[\Delta^l_N(\theta) - \Delta^l_N(\theta)]}{\lambda+\mu+\eta+r} \geq \frac{\underline{\nu}_l}{\lambda+\mu+\eta+r}.\end{aligned}$$

As a result, $\{(\Delta^l_N, \Phi^l_N)\}_N$ is a sequence of uniformly bounded, Lipschitz continuous functions with the same constant. Therefore, it has a convergent sub-sequence.

Now pick $l' = l+1$, and the sub-sequence that $\{(\Delta_N^l, \Phi_N^l)\}_N$ converged. Then we can apply the same argument to show that $\{(\Delta_N^{l'}, \Phi_N^{l'})\}_N$ has a convergent sub-sequence. Since it is a sub-sequence of the sub-sequence that converged for l, then it converges in

By following this process, we can construct a sub-sequence of $\{[\underline{\nu}_N, \bar{\nu}_N]\}_N$ that converges uniformly in every truncation $[\underline{\nu}_l, \bar{\nu}_l]$. Since any compact set $A \subset \mathbb{R}$ of the real numbers is bounded, we have that $A \subset [\underline{\nu}_l, \bar{\nu}_l]$ for l large enough. Therefore, $\{[\underline{\nu}_N, \bar{\nu}_N]\}_N$ converges uniformly in A. Which concludes the proof. □

Constructing the equilibrium: Consider a sequence $\{(\Delta_N, \Phi_N)\}_N$ of fixed points of T_N defined as before on the intervals $\{[\underline{\nu}_N, \bar{\nu}_N]\}_N$, and, passing to a sub-sequence if necessary, let (Δ^*, Φ^*) be its limit.

Let us define $\{bid^*, ask^*, \Delta^*, \phi_o^*, \phi_n^*, s^*\}$ in the following way.

1. Let $\Phi_o^* = \tilde{\Phi}^*$ as defined in equation (35). Since Φ_o^* has bounded variation, it has a density. Then, let ϕ_o^* be it is density.

2. Let $\Phi_n = F - \tilde{\Phi}^*$. Again, due to bounded variation, Φ_n has a density. Let ϕ_n^* be it is density.

3. Let $s^* = \lim_{\nu \nearrow \infty} \sum_\alpha \Phi^*(\alpha, \nu)$.

4. Let bid^* and ask^* be optimal bid and ask function given $\Phi_o^* = \tilde{\Phi}^*$ and $\Phi_n = F - \tilde{\Phi}^*$.

Verifying the equilibrium conditions: To verify the equilibrium conditions, the only thing we need to show is that the optimal bid and ask functions in the sequence $\{(bid_N, ask_N)\}$ associated with $\{(\Delta_N, \Phi_N)\}_N$, converge to the optimal bid and ask functions in the limit bid^* and ask^*. If that is true, then trade probabilities converge and we get all the other equilibrium conditions.

To show that the bid and ask functions converge, we can apply Lemma 12 with a modification. In Lemma 12, the support of ν was compact; now it is not. The reason we can still apply is twofold. First, the sequence uniformly for each compact subset. Second, because the $\int \nu^2 dF$ is bounded, the tail of the distribution goes to zero sufficiently fast so bid and ask functions in a compact set must be bounded. That is, pick a compact set of types $A \subset \Theta$ and consider an owner deciding on an ask price in a meeting with a non-owner with expertise α. If any of the owners in the set A asks his reservation value plus one the lowest profit he can make is $\inf_\theta \int_{\Delta(\theta)}^\infty m_n(u;\alpha)d$. Since the densities are bounded away from zero, the lowest profit he can make doing this bid is bounded away from zero. We know that the profit of a particular ask is bounded above by $ask \int_{ask}^\infty M_n^*(d\nu;\alpha)$. Since $\int \nu^2 dF$ is bounded, $\lim_{ask \nearrow \infty} ask \int_{ask}^\infty M_n^*(d\nu;\alpha) = 0$. Therefore, there is $\bar{ask}$ such that $\inf_\theta \int_{\Delta(\theta)}^\infty m_n(u;\alpha)d > ask \int_{ask}^\infty M_n^*(d\nu;\alpha)$ for all $ask \geq \bar{ask}$.

The $\bar{ask}$ creates a bound on the ask price that allows us to apply Lemma 12 on the set A. Then trade probabilities converge in this set and we get all the other equilibrium conditions for this set. Since we can do this for any compact set, we get the compact convergence of all equilibrium object.

The equations that defined the operator T, and therefore Δ^* and Φ^*, are the same as the equilibrium equations. Since we argued that the bid and ask functions are indeed optimal, all the equilibrium equations are satisfied. Which concludes the proof.

A.4 Private Information and Market Structure

In this section we prove Lemmas 3 through 6, Propositions 2 through 9, and an additional useful result in Lemma 20.

Proof of Lemma 3. Consider any α and $\nu_1 > \nu_0$. Let us show that $\Delta(\theta_1) = \Delta(\alpha, \nu_1) > \Delta(\alpha, \nu_0) = \Delta(\theta_0)$. Equation (7) implies that

$$\Delta(\theta_1) - \Delta(\theta_0) = \frac{\nu_1 - \nu_0 + \lambda(1-s)[\pi_o(\theta_1) - \pi_o(\theta_0)] + \lambda s[\pi_n(\theta_0) - \pi_n(\theta_1)]}{r + \mu + \eta}.$$

Suppose by the way of contradiction that $\Delta(\theta_1) - \Delta(\theta_0) \leq 0$. Since we know that $\nu_1 - \nu_0 > 0$, we must then have that either $\pi_o(\theta_1) - \pi_o(\theta_0) < 0$ or $\pi_n(\theta_0) - \pi_n(\theta_1) < 0$. Neither of these inequalities can happen. An owner of type θ_1 can always mimic the ask price of the owner of type θ_0, and accept the same bids (under private information, under complete information profits are zero for both unless the owner is making the TIOLI offer). By doing so, for any meeting that the owner of type θ_0 sells the asset at price p and makes a profit $p - \Delta(\theta_0)$, the owner of type θ_1 makes a profit $p - \Delta(\theta_1) \geq p - \Delta(\theta_0)$ because $\Delta(\theta_1) - \Delta(\theta_0) \leq 0$. Therefore, we must have that $\pi_o(\theta_1) \geq \pi_o(\theta_0)$.

Similarly, the non-owner of type θ_0 can always mimic the bid price of the non-owner of type θ_1, and accept the same offers (again under private information, under complete information profits are zero for both unless the non-owner is making the TIOLI offer). By doing so, for any meeting that the non-owner of type θ_1 buys the asset at price p and makes a profit $\Delta(\theta_1) - p$, the owner of type θ_0 makes a profit $\Delta(\theta_0) - p \geq \Delta(\theta_1) - p$ because $\Delta(\theta_1) - \Delta(\theta_0) \leq 0$. Therefore, we must have that $\pi_n(\theta_0) \geq \pi_n(\theta_1)$.

Now we have a contradiction since $\Delta(\theta_1) - \Delta(\theta_0) \leq 0$ implies at the same time that either $\pi_o(\theta_1) - \pi_o(\theta_0) < 0$ or $\pi_n(\theta_0) - \pi_n(\theta_1) < 0$, and that $\pi_o(\theta_1) - \pi_o(\theta_0) \geq 0$ or $\pi_n(\theta_0) - \pi_n(\theta_1) \geq 0$. So we have that $\Delta(\theta_1) - \Delta(\theta_0) > 0$.

Similarly, Δ has to be continuous in ν. As before, equation (7) implies that

$$\Delta(\theta_1) - \Delta(\theta_0) = \frac{\nu_1 - \nu_0 + \lambda(1-s)[\pi_o(\theta_1) - \pi_o(\theta_0)] + \lambda s[\pi_n(\theta_0) - \pi_n(\theta_1)]}{r + \mu + \eta}.$$

The previous argument, and the conclusion that $\Delta(\theta_1) - \Delta(\theta_0) > 0$, implies that $\pi_o(\theta_1) -$

$\pi_o(\theta_0) \leq 0$ and $\pi_n(\theta_0) - \pi_n(\theta_1) \leq 0$. But then

$$0 < \Delta(\theta_1) - \Delta(\theta_0) \leq \frac{\nu_1 - \nu_0}{r + \mu + \eta},$$

and we can conclude that $\lim_{\nu_0 \nearrow \nu_1} \Delta(\theta_0) = \Delta(\theta_1)$.

To see that $\lim_{\nu \nearrow \infty} \Delta(\alpha, \nu) = \infty$ note that, because $\Delta(\alpha, \nu)$ is monotone ν, the limit exists. Suppose by the way of contradiction that the limit is not infinity and instead is some real number $\bar{D}$. We know that

$$\Delta(\alpha, \nu) = \frac{\nu + \lambda(1-s)\pi_o(\alpha, \nu) - \lambda s \pi_n(\alpha, \nu)}{r + \mu + \eta}.$$

The profit function $\pi_n(\alpha, \nu)$ is bounded above by $\bar{\Delta} - 0$—there is no investor with reservation value below zero who hold asset in equilibrium for him to buy from. Then we have that

$$\lim_{\nu \nearrow \infty} \Delta(\alpha, \nu) = \lim_{\nu \nearrow \infty} \frac{\nu + \lambda(1-s)\pi_o(\alpha, \nu) - \lambda s \pi_n(\alpha, \nu)}{r + \mu + \eta} \geq \frac{\lim_{\nu \nearrow \infty} \nu - \bar{D}}{r + \mu + \eta} = \infty,$$

which is a contradiction since we assumed that $\lim_{\nu \nearrow \infty} \Delta(\alpha, \nu) = \bar{D}$. An analogous argument can be used to show that $\lim_{\nu \searrow \infty} \Delta(\alpha, \nu) = -\infty$ so we omit it here.

Lemma 20. *Consider a symmetric steady-state equilibrium $\{bid_n, ask_o, \Delta, \Phi_o, \Phi_n, s\}$, and let types $\theta = (\alpha, \nu)$ and $\hat{\theta} = (\hat{\alpha}, \hat{\nu})$ satisfy $\Delta(\theta) = \Delta(\hat{\theta})$ and $\alpha > \hat{\alpha}$. Then the probability of trade of an owner and non-owner satisfy, (i) $\bar{q}_o(\theta) > \bar{q}_o(\hat{\theta})$, and (ii) $\bar{q}_n(\theta) \geq \bar{q}_n(\hat{\theta})$ with strict inequality if $\Delta(\theta) = \Delta(\hat{\theta}) > 0$.*

Proof. First consider the owner's probability to sell an asset in a meeting. Since both investors have the same reservation value, the accept the same bids and ask the same prices when selling the asset. The only difference is the frequency in which they have information when asking a price, which is determined by α and $\hat{\alpha}$ As a result, from equations (9) and (11), we have that

$$\begin{aligned}
\bar{q}_o(\theta) - \bar{q}_o(\hat{\theta}) = & \int \cancel{\mathbb{1}_{\{\Delta_n \geq \Delta(\theta)\}}} - \cancel{\mathbb{1}_{\{\Delta_n \geq \Delta(\hat{\theta})\}}} d\Phi_n(\theta_n) \\
& - \int \xi_o(1-\alpha)\mathbb{1}_{\{ask_o > \Delta_n \geq \Delta(\theta)\}} - \xi_o(1-\hat{\alpha})\mathbb{1}_{\{\hat{ask}_o > \Delta_n \geq \Delta(\hat{\theta})\}} d\Phi_n(\theta_n) \\
& - \int \cancel{\xi_n(1-\alpha_n)\mathbb{1}_{\{\Delta_n \geq \Delta(\theta) > bid_n\}}} - \cancel{\xi_n(1-\alpha_n)\mathbb{1}_{\{\Delta_n \geq \Delta(\hat{\theta}) > bid_n\}}} d\Phi_n(\theta_n) \\
= & \ \xi_o(\alpha - \hat{\alpha}) \int \mathbb{1}_{\{ask_o > \Delta_n \geq \Delta(\theta)\}} d\Phi_n(\theta_n).
\end{aligned}$$

By assumption we know that $\xi_o > 0$ and $\alpha - \hat{\alpha} > 0$, so we just have to show now that $\int \mathbb{1}_{\{ask_o > \Delta_n \geq \Delta(\theta)\}} d\Phi_n(\theta_n) > 0$.

Equation (8) implies that $\phi_n(\tilde{\theta})$ is bounded below by $\frac{\mu f(\tilde{\theta})}{\lambda + \mu + \eta}$ for all $\tilde{\theta}$. Moreover, ask_o has to be strictly bigger than $\Delta(\theta)$. Otherwise, profits would be at most zero. That cannot happen since an ask of $ask = \Delta(\alpha, \nu + \epsilon)$ implies a profit of at least

$$[\Delta(\alpha, \nu + \epsilon) - \Delta(\theta)] \int_{\nu + \epsilon}^{\infty} \phi_n(\alpha, \tilde{\nu}) d\tilde{\nu},$$

which is strictly positive since, by Lemma 3, Δ is strictly increasing in ν, and, by equation (8), $\phi_n(\alpha, \tilde{\nu}) > 0$.

So the ask price has to be strictly above $\Delta(\theta)$. Therefore, $\int \mathbb{1}_{\{ask_o > \Delta_n \geq \Delta(\theta)\}} d\Phi_n(\theta_n)$ is strictly positive and we can conclude that $\bar{q}_o(\theta) > \bar{q}_o(\hat{\theta})$.

The proof for the non-owner's case follow a similar logic and we omit it here. The only difference is that, in equilibrium, ϕ_o is zero when $\Delta(\theta) < 0$. This implies that non-owners with reservation value below zero all have zero probability to buy an asset. That is why in this case the strict inequality only holds when $\Delta(\theta) = \Delta(\hat{\theta}) > 0$. □

Proof of Lemma 4. If $\Delta(\theta) = \Delta(\theta) < 0$, then in a stationary equilibrium we must have that $\phi_o(\theta) = 0$ and $\bar{q}_n(\theta) = 0$. That is, there is no owner with negative reservation value holding assets in equilibrium because they would not issue new assets nor buy from investors with non-negative reservation value. So any existing assets would mature and disappear. This implies that $c(\theta) = c(\hat{\theta}) = 0$ when $\Delta(\theta) = \Delta(\hat{\theta}) < 0$.

For $\Delta(\theta) = \Delta(\theta) \geq 0$ we have the following. Replacing the equilibrium condition of ϕ_o and ϕ_n in (15) we obtain

$$c(\theta) = \frac{\lambda}{2Vol} \times \frac{[\eta + \lambda \bar{q}_n(\theta)]\bar{q}_o(\theta) + [\mu + \lambda \bar{q}_o(\theta)]\bar{q}_n(\theta)}{\eta + \mu + \lambda[\bar{q}_o(\theta) + \bar{q}_n(\theta)]},$$

It is easy to verify that $\frac{[\eta+\lambda\bar{q}_n(\theta)]\bar{q}_o(\theta)+[\mu+\lambda\bar{q}_o(\theta)]\bar{q}_n(\theta)}{\eta+\mu+\lambda[\bar{q}_o(\theta)+\bar{q}_n(\theta)]}$ is strictly increasing in $\bar{q}_o(\theta)$ and $\bar{q}_n(\theta)$, and, from Proposition 20, we know that $\bar{q}_o(\theta) > \bar{q}_o(\hat{\theta})$ and $\bar{q}_n(\theta) \geq \bar{q}_n(\hat{\theta})$. Therefore, we conclude that $c(\theta) > c(\hat{\theta})$.

Proof of Proposition 2. It is easy to see that the most central investor cannot satisfy $\Delta(\theta^*) < 0$ since that would imply $c(\theta^*) = 0$. Then, we must $\Delta(\theta^*) \geq 0$. Suppose by the way of contradiction that α^* does not equal α_I, so it is some $\alpha^* < \alpha_I$. From Lemma 3, we know that $\lim_{\nu \searrow -\infty} \Delta(\alpha_I, \nu) = -\infty$ and $\lim_{\nu \nearrow \infty} \Delta(\alpha_I, \nu) = \infty$. As a result, we can find ν_L and ν_H such that $\Delta(\alpha_I, \nu_L) < \Delta(\theta^*) < \Delta(\alpha_I, \nu_L)$. Since $\Delta(\alpha_I, \nu)$ is continuous ν, then it must exist ν' such that $\Delta(\alpha_I, \nu') = \Delta(\theta^*)$. Then, by Proposition 4, we would have $c(\theta') > c(\theta^*)$ – a contradiction – which concludes the proof.

Proof of Proposition 3. We can consider two cases. If $\sup_{\theta\in\Theta}\{c(\theta)\}$ is strictly greater than $\sup_{\theta\in\Theta}\{c(\theta);\ s.t.\ \alpha \leq \alpha_{I-1}\}$, the results is trivial. For any θ, $c(\theta) \geq \underline{c}$ implies that $c(\theta) > \sup_{\theta\in\Theta}\{c(\theta);\ s.t.\ \alpha \leq \alpha_{I-1}\}$. Therefore, by the definition of sup, we cannot have $\alpha \leq \alpha_{I-1}$. If $\sup_{\theta\in\Theta}\{c(\theta)\} = \sup_{\theta\in\Theta}\{c(\theta);\ s.t.\ \alpha \leq \alpha_{I-1}\}$, then $c(\theta) \geq \underline{c}$ implies that $c(\theta) = \sup_{\theta\in\Theta}\{c(\theta);\ s.t.\ \alpha \leq \alpha_{I-1}\}$. But this is a contradiction since, by Proposition 4, there would exist θ' such that $\alpha' = 1$, $\Delta(\theta') = \Delta(\theta)$ and $c(\theta') > c(\theta) \geq \underline{c} = \sup_{\theta\in\Theta}\{c(\theta)\}$.

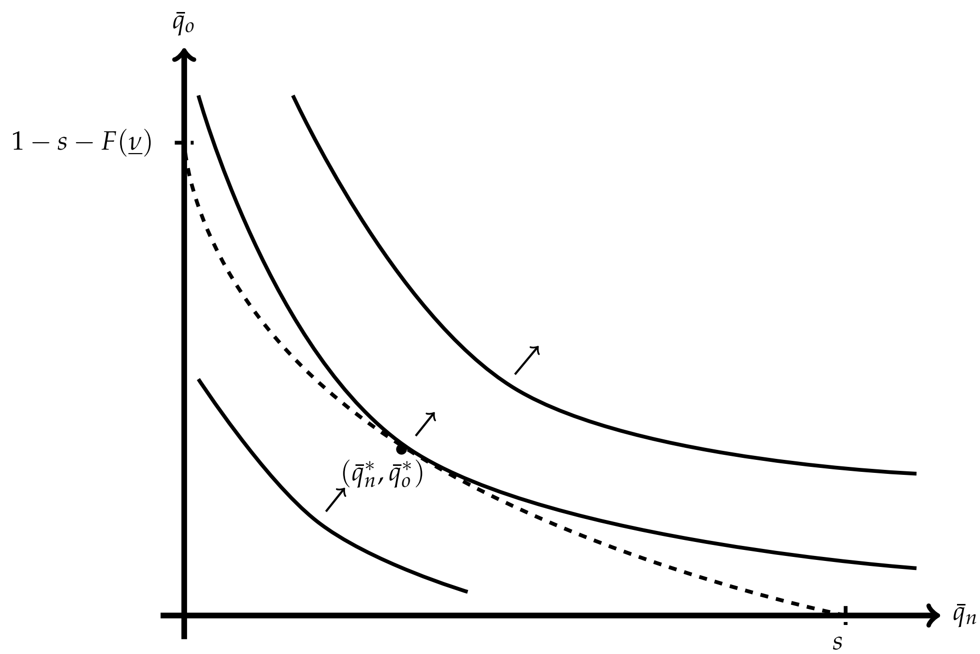
Proof of Propositions 4 and 5. Under complete information, the ex-ante probabilities of buying and selling are an explicit function of the endogenous distributions Φ_o and Φ_n. A non-owner of type ν buys the asset from all owners with type below ν and an owner with type ν sells the asset to all non-owners above ν. Therefore, $\bar{q}_n(\nu) = \Phi_o(\nu)$ and $\bar{q}_o(\nu) = 1 - s - F(\nu) + \Phi_o(\nu) = 1 - s - F(\nu) + \bar{q}_n(\nu)$, where we omit the argument α in Φ_o and Φ_n since all investors have $\alpha = 1$, and where the second equation follows from using the feasibility condition. Combining these two equations provide the set of feasible $\{\bar{q}_n, \bar{q}_o\}$: $\bar{q}_o = Q(\bar{q}_n) \equiv 1 - s - F(\Phi_o^{-1}(\bar{q}_n)) + \bar{q}_n$. Given the equilibrium properties of Φ_o and F, we have that $Q(0) = 1 - s - F(\underline{\nu})$, where $\Delta(\underline{\nu}) = 0$, $Q(s) = 0$. Notice that the point $Q(s) = 0$ is attained in the limit as ν approaches infinity. The dashed line in Figure 14 represents the feasibility equation, $q_o = Q(\bar{q}_n)$.

We can follow a similar approach for centrality. For investors with $\nu < \underline{\nu}$, $C = 0$. And, using equation (8) together with the definition of centrality we obtain

$$C(\bar{q}_n, \bar{q}_o) = \frac{\lambda}{2Vol} \frac{[\eta + \lambda \bar{q}_n]\bar{q}_o + [\mu + \lambda \bar{q}_o]\bar{q}_n}{\mu + \eta + \lambda[\bar{q}_o + \bar{q}_n]} \text{ for any investor with } \nu \geq \underline{\nu} \; .$$

This equation gives downward sloping centrality level curves in the space of $(\bar{q}_n, \bar{q}_o)$. These level curves are represented as solid lines in Figure 14. Intuitively, an investor's centrality increases if they have a higher $\bar{q}_o$ or $\bar{q}_n$.

Figure 14: Equilibrium pairs $\bar{q}_n$ and $\bar{q}_o$ and centrality level curves



Notes: The figure presents the equilibrium feasibility set $\bar{q}_0 = Q(\bar{q}_n)$, and the centrality level curves $\tilde{c} = C(\bar{q}_n, \bar{q}_o)$, for different centrality levels $\tilde{c}$. The point $\{\bar{q}_n^*, \bar{q}_o^*\}$ represents the most central investor.

Now consider the following problem

$$(\bar{q}_n, \bar{q}_o) = \max_{(\bar{q}_n, \bar{q}_o)} \left\{ C(\bar{q}_n, \bar{q}_o); \text{ subject to } \bar{q}_o = Q(\bar{q}_n) \text{ and } \bar{q}_n \in [0, s] \right\}. \tag{44}$$

The solution to problem 44 exists because the set of $(\bar{q}_n, \bar{q}_o)$ satisfying $\bar{q}_o = Q(\bar{q}_n)$ and $\bar{q}_n \in [0, s]$ is compact, and the function $C(\bar{q}_n, \bar{q}_o)$ is continuous in this set.

It is easy to see the $C(\bar{q}_n, \bar{q}_o)$ is differentiable in both $\bar{q}_o$ and $\bar{q}_n$. If in addition $Q(\bar{q}_n)$ is differentiable for $\bar{q}_n \in [0, s]$, then we can replace $\bar{q}_o = Q(\bar{q}_n)$ in $C(\bar{q}_n, \bar{q}_o)$ and then optimize by using the first order condition. Later, we show that the solution must be interior, satisfying $\bar{q}_n \in (0, s)$.

The function $Q(\bar{q}_n) = 1 - s - F(\Phi_o^{-1}(\bar{q}_n)) + \bar{q}_n$ is differentiable if Φ_o is differentiable, and the derivative bounded away from zero. Given that $\bar{q}_n(\nu) = \Phi_o(\nu)$ and $\bar{q}_o(\nu) = 1 - s - F(\nu) + \Phi_o(\nu)$, $\bar{q}_o$ and $\bar{q}_n$ are continuous in ν. Further, since $\Phi_o(\nu) = \int_{-\infty}^{\nu} \phi_o(\tilde{\nu}) d\tilde{\nu}$ and, from (8) we have that $\phi_o(\nu) = \frac{\eta + \lambda\bar{q}_n(\nu)}{\eta + \mu + \lambda\bar{q}_o(\nu) + \lambda\bar{q}_n(\nu)} f(\nu)$, we obtain that $\Phi_o(\nu)$ is differentiable. Thus, $Q(\bar{q}_n)$ is differentiable.

We can then replace the above expressions into the definition of $Q(\bar{q}_n)$ and obtain that

$$Q'(\bar{q}_n) = 1 - \frac{f(\Phi_o^{-1}(\bar{q}_n))}{\phi_o(\Phi_o^{-1}(\bar{q}_n))} = 1 - \frac{\eta + \mu + \lambda\bar{q}_o + \lambda\bar{q}_n}{\eta + \lambda\bar{q}_n} = -\frac{\mu + \lambda\bar{q}_o}{\eta + \lambda\bar{q}_n}.$$

Similarly, replacing $\bar{q}_o = Q(\bar{q}_n)$ in $C(\bar{q}_n, \bar{q}_o)$ and defining $\tilde{C}(\bar{q}_n) \equiv \frac{2Vol}{\lambda} C(\bar{q}_n, Q(\bar{q}_n))$ provides

$$\tilde{C}'(\bar{q}_n) = \frac{2Vol}{\lambda} \left[\frac{\partial C(\bar{q}_n, \bar{q}_o)}{\partial \bar{q}_n} + \frac{\partial C(\bar{q}_n, \bar{q}_o)}{\partial \bar{q}_o} \times Q'(\bar{q}_n) \right].$$

Note that

$$\begin{aligned}
\frac{\lambda}{2Vol} \frac{\partial C(\bar{q}_n, \bar{q}_o)}{\partial \bar{q}_n} &= \frac{[\mu + 2\lambda\bar{q}_o]\{\mu + \eta + \lambda[\bar{q}_o + \bar{q}_n]\} - \{[\eta + \lambda\bar{q}_n]\bar{q}_o + [\mu + \lambda\bar{q}_o]\bar{q}_n\}\lambda}{\{\mu + \eta + \lambda[\bar{q}_o + \bar{q}_n]\}^2} \\
&= \frac{\mu\lambda\bar{q}_n + [\mu + 2\lambda\bar{q}_o]\{\mu + \eta + \lambda\bar{q}_o\} - \{\eta\bar{q}_o + \mu\bar{q}_n\}\lambda}{\{\mu + \eta + \lambda[\bar{q}_o + \bar{q}_n]\}^2} \\
&= \frac{[\mu + 2\lambda\bar{q}_o]\{\mu + \eta + \lambda\bar{q}_o\} - \eta\bar{q}_o\lambda}{\{\mu + \eta + \lambda[\bar{q}_o + \bar{q}_n]\}^2} = \frac{[\mu + \lambda\bar{q}_o]\{\mu + \eta + \lambda\bar{q}_o\} + \lambda\bar{q}_o\{\mu + \lambda\bar{q}_o\}}{\{\mu + \eta + \lambda[\bar{q}_o + \bar{q}_n]\}^2} \\
&= [\mu + \lambda\bar{q}_o] \frac{\mu + \eta + 2\lambda\bar{q}_o}{\{\mu + \eta + \lambda[\bar{q}_o + \bar{q}_n]\}^2},
\end{aligned}$$

and,

$$\frac{\lambda}{2Vol} \frac{\partial C(\bar{q}_n, \bar{q}_o)}{\partial \bar{q}_o} = [\eta + \lambda\bar{q}_n] \frac{\mu + \eta + 2\lambda\bar{q}_n}{\{\mu + \eta + \lambda[\bar{q}_o + \bar{q}_n]\}^2}.$$

Then,

$$\begin{aligned}
\tilde{C}'(\bar{q}_n) &= \frac{2Vol}{\lambda} \left[\frac{\partial C(\bar{q}_n, \bar{q}_o)}{\partial \bar{q}_n} + \frac{\partial C(\bar{q}_n, \bar{q}_o)}{\partial \bar{q}_o} \times Q'(\bar{q}_n) \right] \\
&= [\mu + \lambda\bar{q}_o] \frac{\mu + \eta + 2\lambda\bar{q}_o}{\{\mu + \eta + \lambda[\bar{q}_o + \bar{q}_n]\}^2} - [\eta + \lambda\bar{q}_n] \frac{\mu + \eta + 2\lambda\bar{q}_n}{\{\mu + \eta + \lambda[\bar{q}_o + \bar{q}_n]\}^2} \frac{\mu + \lambda\bar{q}_o}{\eta + \lambda\bar{q}_n} \\
&= \frac{2\lambda[\mu + \lambda\bar{q}_o]}{\{\mu + \eta + \lambda[\bar{q}_o + \bar{q}_n]\}^2} (\bar{q}_o - \bar{q}_n).
\end{aligned}$$

Because in the interior the maximum requires $\tilde{C}'(\bar{q}_n) = 0$, in the interior the maximum is attained at $\bar{q}_o = Q(\bar{q}_n) = \bar{q}_n$.

We now show that centrality is not maximized in the corners. Given that in the interior we found a candidate solution satisfying $\tilde{C}'(\bar{q}_n) = 0$, it suffices to show that $\tilde{C}'(0) > 0$ and $\tilde{C}'(s) < 0$. Because $Q(\bar{q}_n)$ is differentiable for $\bar{q}_n \in [0, s]$, it is also continuous. For $\bar{q}_n = 0$ we have that

$Q(0) = 1 - s - F(\underline{\nu})$. From the equilibrium equations we obtain that $\dot{s} = \eta[1 - s - F(\underline{\nu})] - \mu s = 0$ which implies that in a stationary equilibrium $s = \frac{\eta}{\mu+\eta}[1 - F(\underline{\nu})]$. In turn, this implies that $Q(0) = 1 - s - F(\underline{\nu}) = \frac{\mu}{\mu+\eta}[1 - F(\underline{\nu})] > 0$, so that $\tilde{C}'(0) > 0$. Since the derivative is strictly positive at the lower bound then you can increase the obj function by increasing $\bar{q}_n$ so the lower bound cannot be a solution. When $\bar{q}_n = s$ we have that

$$Q(s) = 1 - s - \lim_{\tilde{s} \nearrow s} F(\Phi_o^{-1}(\tilde{s})) + s = 1 - s - \lim_{\nu \nearrow \infty} F(\nu) + s = 1 - s - 1 + s = 0.$$

Then, $\tilde{C}'(s) < 0$. Since the derivative is strictly negative at the upper bound you can increase the obj function by decreasing $\bar{q}_n$ so the upper bound cannot be a solution.

Because $Q(\bar{q}_n) - \bar{q}_n$ is continuous in $\bar{q}_n \in [0, s]$, $Q(0) - 0 > 0$ and $Q(s) - s < 0$, there exists $\bar{q}_n^* \in (0, s)$ such that $\bar{q}_o^* = Q(\bar{q}_n^*) = \bar{q}_n^*$, and since $Q'(\bar{q}_n) = -\frac{\mu+\lambda\bar{q}_o}{\eta+\lambda\bar{q}_n} < 0$, $\bar{q}_n^*$ is unique. Finally, replacing $\bar{q}_o^* = Q(\bar{q}_n^*) = \bar{q}_n^*$ in the definition of $s_n(\nu^*)$ provides

$$s_n(\nu^*) = \left[1 + \frac{\eta + \lambda\bar{q}_n(\nu^*)}{\mu + \lambda\bar{q}_n(\nu^*)}\right]^{-1}.$$

Now we turn to **Proposition 4**. Note also that

$$s_n(1,\nu) = \frac{[\mu + \lambda\bar{q}_o(1,\nu)]\bar{q}_n(1,\nu)}{[\eta \mathbb{1}_{\{\Delta(1,\nu) \geq 0\}} + \lambda\bar{q}_n(1,\nu)]\bar{q}_o(1,\nu) + [\mu + \lambda\bar{q}_o(1,\nu)]\bar{q}_n(1,\nu)}.$$

From the equation above, $s_n(1,\nu)$ is strictly increasing in ν for any ν such that $\Delta(1,\nu) > 0$. That is because in this case,

$$\begin{aligned}
\frac{ds_n(1,\nu)}{d\nu} = & \left\{ \frac{[\mu + \lambda\bar{q}_o(1,\nu)]\{[\eta + \lambda\bar{q}_n(1,\nu)]\bar{q}_o(1,\nu) + [\mu + \lambda\bar{q}_o(1,\nu)]\bar{q}_n(1,\nu)\}}{\{[\eta + \lambda\bar{q}_n(1,\nu)]\bar{q}_o(1,\nu) + [\mu + \lambda\bar{q}_o(1,\nu)]\bar{q}_n(1,\nu)\}^2} \right. \\
& \left. - \frac{[\mu + \lambda\bar{q}_o(1,\nu)]\bar{q}_n(1,\nu)\{\lambda\bar{q}_o(1,\nu) + [\mu + \lambda\bar{q}_o(1,\nu)]\}}{\{[\eta + \lambda\bar{q}_n(1,\nu)]\bar{q}_o(1,\nu) + [\mu + \lambda\bar{q}_o(1,\nu)]\bar{q}_n(1,\nu)\}^2} \right\} \times \frac{d\bar{q}_n(1,\nu)}{d\nu} \\
& + \left\{ \frac{\lambda\bar{q}_n(1,\nu)\{[\eta + \lambda\bar{q}_n(1,\nu)]\bar{q}_o(1,\nu) + [\mu + \lambda\bar{q}_o(1,\nu)]\bar{q}_n(1,\nu)\}}{\{[\eta + \lambda\bar{q}_n(1,\nu)]\bar{q}_o(1,\nu) + [\mu + \lambda\bar{q}_o(1,\nu)]\bar{q}_n(1,\nu)\}^2} \right. \\
& \left. - \frac{[\mu + \lambda\bar{q}_o(1,\nu)]\bar{q}_n(1,\nu)\{[\eta + \lambda\bar{q}_n(1,\nu)] + \lambda\bar{q}_n(1,\nu)\}}{\{[\eta + \lambda\bar{q}_n(1,\nu)]\bar{q}_o(1,\nu) + [\mu + \lambda\bar{q}_o(1,\nu)]\bar{q}_n(1,\nu)\}^2} \right\} \times Q'(\bar{q}_n)\frac{d\bar{q}_n(1,\nu)}{d\nu} \\
= & \underbrace{\left\{ \frac{[\mu + \lambda\bar{q}_o(1,\nu)]\eta\bar{q}_o(1,\nu)}{\{[\eta + \lambda\bar{q}_n(1,\nu)]\bar{q}_o(1,\nu) + [\mu + \lambda\bar{q}_o(1,\nu)]\bar{q}_n(1,\nu)\}^2} \right\} \times \frac{d\bar{q}_n(1,\nu)}{d\nu}}_{>0} \\
& \underbrace{- \left\{ \frac{\mu\bar{q}_n(1,\nu)[\eta + \lambda\bar{q}_n(1,\nu)]}{\{[\eta + \lambda\bar{q}_n(1,\nu)]\bar{q}_o(1,\nu) + [\mu + \lambda\bar{q}_o(1,\nu)]\bar{q}_n(1,\nu)\}^2} \right\} \times Q'(\bar{q}_n)\frac{d\bar{q}_n(1,\nu)}{d\nu}}_{>0}.
\end{aligned}$$

Since $\frac{d\bar{q}_n(1,\nu)}{d\nu} = \phi_o(1,\nu) = \frac{\eta + \lambda\bar{q}_n(1,\nu)}{\eta + \mu + \lambda\bar{q}_o(1,\nu) + \lambda\bar{q}_n(1,\nu)} f(1,\nu) > 0$, and $Q'(\bar{q}_n) = -\frac{\mu+\lambda\bar{q}_o}{\eta+\lambda\bar{q}_n} < 0$, we can conclude that $\frac{ds_n(1,\nu)}{d\nu} > 0$.

If $s_n(1,\nu_a) < s_n(1,\nu_b) \leq s_n(1,\nu^*)$, because $\frac{ds_n(1,\nu)}{d\nu} > 0$, we must have that $\nu^* \geq \nu_b > \nu_a$. Now, from the proof of Proposition 5, we know that

$$\tilde{C}'(\bar{q}_n) = \frac{2\lambda[\mu + \lambda\bar{q}_o]}{\{\mu + \eta + \lambda[\bar{q}_o + \bar{q}_n]\}^2}(\bar{q}_o - \bar{q}_n).$$

We know that $\bar{q}_o(1,\nu^*) = \bar{q}_n(1,\nu^*)$. Therefore, because $\frac{d\bar{q}_n(1,\nu)}{d\nu} > 0$ and $\frac{d\bar{q}_o(1,\nu)}{d\nu} = Q'(\bar{q}_n(1,\nu))\frac{d\bar{q}_n(1,\nu)}{d\nu} < 0$, $\bar{q}_o(1,\nu) > \bar{q}_n(1,\nu)$ for all $\nu \leq \nu^*$ and centrality is increasing in ν. Thus, we conclude that $c(1,\nu_a) < c(1,\nu_b) \leq c(1,\nu^*)$. The opposite argument works if $s_n(1,\nu_a) > s_n(1,\nu_b) \geq s_n(1,\nu^*)$. In this case, $\nu^* \leq \nu_b < \nu_a$ and centrality is decreasing in ν because $\bar{q}_o(1,\nu) < \bar{q}_n(1,\nu)$.

Proof of Propositions 6. To keep the notation short, define $x^a = x(\theta_a)$ and $x^b = x(\theta_b)$ for any function x of θ. That is, $s_n^a = s_n(\theta_a)$, $s_n^b = s_n(\theta_b)$ and so on.

First note that, since $s_n^a = s_n^b \in (0,1)$, we must have that $\bar{q}_o^a, \bar{q}_o^b, \Delta^a, \Delta^b > 0$. That is because the measure of owners with negative reservation value has to be zero otherwise they would just dispose of the asset and s_n would be equal to zero.

Now, suppose by the way of contradiction that $c^b \geq c^a$. From the definition of centrality and using equilibrium equation (8), we have that

$$c^b = \frac{\lambda}{2Vol}\frac{[\eta + \lambda\bar{q}_n^b]\bar{q}_o^b + [\mu + \lambda\bar{q}_o^b]\bar{q}_n^b}{\mu + \eta + \lambda[\bar{q}_o^b + \bar{q}_n^b]} \geq \frac{\lambda}{2Vol}\frac{[\eta + \lambda\bar{q}_n^a]\bar{q}_o^a + [\mu + \lambda\bar{q}_o^a]\bar{q}_n^a}{\mu + \eta + \lambda[\bar{q}_o^a + \bar{q}_n^a]} = c^a\ .$$

The centrality map above is increasing in both $\bar{q}_n$ and $\bar{q}_o$. As a result, for the inequality above to hold it is necessary that one of the three statements hold: *(i)* $\bar{q}_n^b \geq \bar{q}_n^a$ and $\bar{q}_o^b \geq \bar{q}_o^a$, *(ii)* $\bar{q}_n^b > \bar{q}_n^a$ and $\bar{q}_o^b < \bar{q}_o^a$, or *(i)* $\bar{q}_n^b < \bar{q}_n^a$ and $\bar{q}_o^b > \bar{q}_o^a$.

Statement *(i)* is a contradiction. Given that $\alpha_a > \alpha_b$, the only way that we can have $\bar{q}_n^b \geq \bar{q}_n^a$ is if $\Delta^b > \Delta^a$. At the same time, given that $\alpha_a > \alpha_b$, the only way that we can have $\bar{q}_o^b \geq \bar{q}_o^a$ is if $\Delta^b < \Delta^a$. Which is a contradiction because we cannot have $\Delta^b > \Delta^a$ and $\Delta^b < \Delta^a$.

Statements *(ii)* and *(iii)* also imply a contradiction. To see this, note that, from equation (16) we have

$$s_n^a = \frac{[\mu + \lambda\bar{q}_o^a]\bar{q}_n^a}{[\eta + \lambda\bar{q}_n^a]\bar{q}_o^a + [\mu + \lambda\bar{q}_o^a]\bar{q}_n^a} = \frac{[\mu + \lambda\bar{q}_o^b]\bar{q}_n^b}{[\eta + \lambda\bar{q}_n^b]\bar{q}_o^b + [\mu + \lambda\bar{q}_o^b]\bar{q}_n^b} = s_n^b$$

But note that

$$\begin{aligned}
\frac{\partial}{\partial\bar{q}_n}\cdot\frac{[\mu + \lambda\bar{q}_o]\bar{q}_n}{[\eta + \lambda\bar{q}_n]\bar{q}_o + [\mu + \lambda\bar{q}_o]\bar{q}_n} &= \frac{[\mu + \lambda\bar{q}_o]\{[\eta + \lambda\bar{q}_n]\bar{q}_o + [\mu + \lambda\bar{q}_o]\bar{q}_n\} - [\mu + \lambda\bar{q}_o]\bar{q}_n[\mu + 2\lambda\bar{q}_o]}{\{[\eta + \lambda\bar{q}_n]\bar{q}_o + [\mu + \lambda\bar{q}_o]\bar{q}_n\}^2} \\
&= \frac{[\mu + \lambda\bar{q}_o]\eta\bar{q}_o}{\{[\eta + \lambda\bar{q}_n]\bar{q}_o + [\mu + \lambda\bar{q}_o]\bar{q}_n\}^2} > 0, \qquad \text{and} \\
\frac{\partial}{\partial\bar{q}_o}\cdot\frac{[\mu + \lambda\bar{q}_o]\bar{q}_n}{[\eta + \lambda\bar{q}_n]\bar{q}_o + [\mu + \lambda\bar{q}_o]\bar{q}_n} &= \frac{\lambda\bar{q}_n\{[\eta + \lambda\bar{q}_n]\bar{q}_o + [\mu + \lambda\bar{q}_o]\bar{q}_n\} - [\mu + \lambda\bar{q}_o]\bar{q}_n[\eta + 2\lambda\bar{q}_n]}{\{[\eta + \lambda\bar{q}_n]\bar{q}_o + [\mu + \lambda\bar{q}_o]\bar{q}_n\}^2} \\
&= \frac{\lambda\bar{q}_n[\eta + \lambda\bar{q}_n]\bar{q}_o - [\mu + \lambda\bar{q}_o]\bar{q}_n[\eta + \lambda\bar{q}_n]}{\{[\eta + \lambda\bar{q}_n]\bar{q}_o + [\mu + \lambda\bar{q}_o]\bar{q}_n\}^2}
\end{aligned}$$

$$= \frac{-\mu\bar{q}_n[\eta+\lambda\bar{q}_n]}{\{[\eta+\lambda\bar{q}_n]\bar{q}_o+[\mu+\lambda\bar{q}_o]\bar{q}_n\}^2} < 0\,.$$

But then, $\bar{q}_n^b > \bar{q}_n^a$ and $\bar{q}_o^b < \bar{q}_o^a$ would imply $s_n^a < s_n^b$, which is a contradiction. And $\bar{q}_n^b < \bar{q}_n^a$ and $\bar{q}_o^b > \bar{q}_o^a$ would imply $s_n^a > s_n^b$, which is again a contradiction.

Therefore, we cannot have that $c^b \geq c^a$ and must have $c^a > c^b$. Which concludes the proof.

Proof of Lemma 5. Pick any $\theta = (\alpha, \nu)$ such that $\Delta(\theta) > 0$. First, let us consider the case where $\alpha = \alpha_I = 1$. This investor buys an asset with probability one whenever he has the bargaining power and there are gains from trade. In the same way, he sells an asset with probability one whenever he has the bargaining power and there are gains from trade. For any other $\hat{\theta}$, either we have $\Delta(\theta) > \Delta(\hat{\theta})$, $\Delta(\theta) < \Delta(\hat{\theta})$ or $\Delta(\theta) = \Delta(\hat{\theta})$. The measure of cases in which $\Delta(\theta) = \Delta(\hat{\theta})$ is zero. In the other two cases, we have that $q(\theta, \hat{\theta})$ is bounded below by $\xi_o > 0$ and $q(\hat{\theta}, \theta)$ is bounded below by $\xi_n > 0$. So we can conclude that $np(\theta) = 1$.

Now, let us consider the case where $\alpha = \alpha_l = 0$. Then, by Lemma 1, we know that $ask_o(\alpha_l, \Delta(\theta)) > \Delta(\theta)$. Define $\theta_\epsilon = \theta + (\alpha, \nu) = (\alpha, \nu + \epsilon)$. Note that we can take $\bar{\epsilon} > 0$ small enough such that $bid_n(\alpha_l, \Delta(\theta_\epsilon)) < \Delta(\theta)$ for all $\epsilon \in (0, \bar{\epsilon})$. That is because the objective function that defines the bid is continuous and $bid_n(\alpha_l, \Delta(\theta_\epsilon))$ is increasing in the reservation value, which is increasing in ν. As a result, the investor type $\theta = (\alpha, \nu)$ won't sell or buy to any investor type θ_ϵ for $\epsilon \in (0, \bar{\epsilon})$. Since F has positive density everywhere, we then have that $np(\theta) < 1$.

Finally, a similar argument provides $np_{out}(\theta) > np_{out}(\hat{\theta})$ and $np_{in}(\theta) > np_{in}(\hat{\theta})$.

Proof of Proposition 7. This result is straightforward. For every θ in $\bar{\Theta}$, by Proposition 3, we have that $\alpha = 1$. Then, from Lemma 5, we must have $np(\theta) = 1$ for all θ in $\bar{\Theta}$ so

$$\frac{\int_{\theta\in\bar{\Theta}} np(\theta) f(\theta) d\theta}{\int_{\theta\in\bar{\Theta}} f(\theta) d\theta} = \frac{\int_{\theta\in\bar{\Theta}} f(\theta) d\theta}{\int_{\theta\in\bar{\Theta}} f(\theta) d\theta} = 1.$$

Lastly, from 5, we must have $np(\theta) < 1$ for all θ such that $\alpha = 0$. But the set of θ such that $\alpha = 0$ cannot belong to $\bar{\Theta}$. So,

$$\frac{\int_{\theta\notin\bar{\Theta}} np(\theta) f(\theta) d\theta}{\int_{\theta\notin\bar{\Theta}} f(\theta) d\theta} < \frac{\int_{\theta\notin\bar{\Theta}} f(\theta) d\theta}{\int_{\theta\notin\bar{\Theta}} f(\theta) d\theta} = 1 = \frac{\int_{\theta\in\bar{\Theta}} np(\theta) f(\theta) d\theta}{\int_{\theta\in\bar{\Theta}} f(\theta) d\theta}.$$

Proof of Proposition 8. Equation 18 can be written as

$$\begin{aligned}
st(\theta) &= \frac{\phi_o(\theta)}{f(\theta)} \int \frac{\phi_n(\hat{\theta})}{f(\hat{\theta})} q(\theta, \hat{\theta}) dF(\hat{\theta}) + \frac{\phi_n(\theta)}{f(\theta)} \int \frac{\phi_o(\hat{\theta})}{f(\hat{\theta})} q(\hat{\theta}, \theta) dF(\hat{\theta}) \\
&= \frac{\phi_o(\theta)}{f(\theta)} \int q(\theta, \hat{\theta}) d\Phi_n(\hat{\theta}) + \frac{\phi_n(\theta)}{f(\theta)} \int q(\hat{\theta}, \theta) d\Phi_o(\hat{\theta}) \\
&= \frac{\phi_o(\theta)\bar{q}_o(\theta)}{f(\theta)} + \frac{\phi_n(\theta)\bar{q}_n(\theta)}{f(\theta)} = \frac{2Vol}{\lambda} c(\theta),
\end{aligned}$$

where the first and second equalities are just from rewriting the equation, the third equality is from applying equations 9 and 9, and the fourth equality comes from the definition of centrality given by equation 15.

Proof of Proposition 9. Suppose by the way of contradiction that the result does not hold. Then, we either have $V_o(\theta) \geq V_o(\hat{\theta})$ or $V_n(\theta) \geq V_n(\hat{\theta})$, or both.

Assume that $V_o(\theta) \geq V_o(\hat{\theta})$. From equation (5), this implies that $\pi_o(\theta) \geq \pi_o(\hat{\theta})$. Since $\hat{\alpha} > \alpha$, investor type θ can only have a higher profit when selling the asset if he has a lower reservation value for it. That is, $\Delta(\theta) < \Delta(\hat{\theta})$. Given that the investor with higher expertise have an advantage when selling the asset and $\Delta(\hat{\theta}) > \Delta(\theta)$, we then have that $V_n(\hat{\theta}) > V_n(\theta) \Leftrightarrow -V_n(\theta) > -V_n(\hat{\theta})$. Adding up the two inequalities $V_o(\theta) \geq V_o(\hat{\theta})$ and $-V_n(\theta) > -V_n(\hat{\theta})$ we obtain that

$$\Delta(\theta) = V_o(\theta) - V_n(\theta) > V_o(\hat{\theta}) - V_n(\hat{\theta}) = \Delta(\hat{\theta}).$$

Which contradicts our previous conclusion that $\Delta(\theta) < \Delta(\hat{\theta})$. The same argument applies for the case that $V_n(\theta) \geq V_n(\hat{\theta})$.

Proof of Lemma 6. We can write $\pi_o(\theta) - \pi_o(\hat{\theta})$ from (3) as

$$\begin{aligned}\pi_o(\theta) - \pi_o(\hat{\theta}) &= (\alpha - \hat{\alpha}) \int \left(\tilde{\Delta}_n - \Delta_o\right) \mathbb{1}_{\{\tilde{\Delta}_n \geq \Delta_o\}} - \left(ask_o - \Delta_o\right) \mathbb{1}_{\{\tilde{\Delta}_n \geq ask_o\}} d\frac{\Phi_n(\tilde{\theta})}{1-s} \\ &= (\alpha - \hat{\alpha}) \int \left(\tilde{\Delta}_n - ask_o\right) \mathbb{1}_{\{\tilde{\Delta}_n \geq ask_o\}} + \left(\tilde{\Delta}_n - \Delta_o\right) \mathbb{1}_{\{ask_o > \tilde{\Delta}_n \geq \Delta_o\}} d\frac{\Phi_n(\tilde{\theta})}{1-s} > 0.\end{aligned}$$

Therefore, $\alpha > \hat{\alpha}$ implies $\pi_o(\theta) > \pi_o(\hat{\theta})$. The argument for π_n is analogous. The only difference for π_n is that, in equilibrium, ϕ_n is zero when $\Delta(\theta) < 0$. This implies that non-owners with reservation value below zero all have zero probability to buy an asset. That is why the strict inequality only holds when $\Delta(\theta) = \Delta(\hat{\theta}) > 0$.

A.5 Extension with 13-F investors

In this section we provide a formal derivation of the results discussed in Section 6.2.1. As a reminder, the distribution of types across 13-F and non-13-F investors is given by $F = \omega_{13}F_{13} + \omega_{n13}F_{n13}$. For a given filing window $[T_0, T_0 + T]$, each 13-F investor draws an initial date for the filing window, $T_0 \geq t_0$, with Poisson arrival $\gamma > 0$. Initial filing dates are independent from each other and are not known beforehand. The information revealed by filing a 13-F is imperfect. In a meeting after a 13-F investor files, $\rho \in (0, 1]$ is the probability that the report perfectly reveals the type of the 13-F investor to the counterparty. This shock is independent and identically distributed across 13-F investors and meetings and is independent from other shocks.

An equilibrium is defined in the same way as the equilibrium definition in 1, but adjusting for the fact that the objects are time dependent instead of been defined in steady state. There is also an extra equilibrium object which is the choice of a filing date $\tilde{T}_0$ given a filing window $[T_0, T_0 + T]$. We find that the optimal filing date is $\tilde{T}_0 \equiv T_0 + T$ for all investors. And that strategy is strictly better than picking any other filing date for all investors with positive reservation value—that is, investors who trade with strictly positive probability.

Proof of Proposition 10. The result regarding the delay choice in Proposition 10 is straightforward and we just provide a sketch here. Filing Form 13-F cannot make the investor better off, as truthfully revealing her type in a meeting is always an option. Moreover, if she has strictly positive reservation value, then she loses the information gains that she can obtain when her trade counterparty sets up a price under private information.

Given a delay T, we now consider how a filing affects the 13-F investor's conditional probability of trade with a given set of counterparties, $\hat{\Theta} \subseteq \Theta$. The conditional probability the 13-F investor trades with counterparties in $\hat{\Theta}$ at time t is given by

$$\bar{q}_t^{13F}(\theta; \hat{\Theta}) = \omega_o \, \bar{q}_{ot}^{13F}(\theta; \hat{\Theta}) + \omega_n \, \bar{q}_{nt}^{13F}(\theta; \hat{\Theta}), \tag{45}$$

where $\bar{q}_{ot}^{13F}(\theta; \hat{\Theta})$ and $\bar{q}_{nt}^{13F}(\theta; \hat{\Theta})$ are the conditional probabilities the 13-F investor sells an asset to and buys an asset from a counterparty in $\hat{\Theta}$, respectively, and $\omega_{o,t} = \phi_{o,t}(\theta)/f(\theta)$ and $\omega_{n,t} = 1 - \omega_{o,t}$ are weights that give the probability of being an owner or non-owner, respectively. We omit the expressions for $\bar{q}_{ot}^{13F}(\theta; \hat{\Theta})$ and $\bar{q}_{nt}^{13F}(\theta; \hat{\Theta})$, but they follow from (9)-(11) replacing the distributions ϕ_n and ϕ_o with conditional distributions with support over $\hat{\Theta}$.

It is useful to define the discontinuity in trade probability for an investor of type θ trading with a group of investors $\hat{\Theta}$ as $D_t^{13F}(\theta; \hat{\Theta}) = \lim_{\epsilon \searrow 0} \bar{q}_{t+\epsilon}^{13F}(\theta, \hat{\Theta}) - \bar{q}_{t-\epsilon}^{13F}(\theta; \hat{\Theta})$. We are interested in the effect of a 13-F filing on this discontinuity. That is, consider a investor who received a Poisson shock to file the 13-F, and chose a delay $\tilde{t} = T$. In this case, we can write the discontinuity in trade probability in closed form as,

$$\begin{aligned} D_T^{13F}(\theta; \hat{\Theta}) = \; & \omega_o \xi_n \mathbb{E}_{\hat{\theta} \in \hat{\Theta}} \Big\{ \rho(1 - \alpha_n) \mathbb{1}_{\{\hat{\Delta}_n \geq \Delta(\theta) > bid_T^{13F}(\hat{\Delta}_n; \alpha)\}} \Big\} \\ & + \omega_n \xi_o \mathbb{E}_{\hat{\theta} \in \hat{\Theta}} \Big\{ \rho(1 - \alpha_o) \mathbb{1}_{\{ask_T^{13F}(\hat{\Delta}_o; \alpha) > \Delta(\theta) \geq \hat{\Delta}_o\}} \Big\}. \end{aligned} \tag{46}$$

The expectation operator in the first and second term of (46) are conditional expectations over $\theta_n \in \hat{\Theta}$ and $\theta_o \in \hat{\Theta}$, respectively. The effect of a 13-F filing depends positively on how informative a filing is, ρ, and negatively on the screening expertise of the set of counterparties, α_n and α_o. Intuitively, if the set of counterparties includes investors with higher screening ability, the discontinuity is lower.

We now prove the following results from Proposition 10: for a 13-F investor who files with delay $\tilde{t} = T$, (i) a 13-F filing causes a strictly-positive jump in the probability of trade with periphery investors, (ii) a 13-F filing causes no change in the probability of trade with core investors, and (iii) as a result of (i) and (ii), a 13-F filing shifts the probability of trade towards periphery investors relative to core investors.

Both results (i) and (ii) follow direct from equation (46), Lemmas 1 and 2 which shows that there is a distortion on bid and ask implied by private information, and Proposition 3 which states that the central traders have $\alpha = \alpha_I = 1$. According to Proposition 3, all investors in Θ^c have $\alpha = \alpha_I = 1$. Therefore, by equation (46), we have to have $D_T^{13F}(\theta; \Theta^c) = 0$. Moreover, all investors with $\alpha < \alpha_I = 1$ are in Θ^p. Therefore, again by equation (46) and using the implied distortion in bid and ask shown in Lemmas 1 and 2, $D_T^{13F}(\theta; \Theta^p) > 0$. Result (iii) follows directly from (i) and (ii).

It is worth mentioning that information revelation can potentially increase the probability of trade with both core and periphery investors if $\alpha_I < 1$. However, if α_I is close to 1, it must have a greater impact on the probability trade with periphery investors relative to core investors precisely because core investors already have an informational advantage. We do not provide the result for $\alpha_I < 1$ to keep the presentation simpler but can provide it under request.

B Appendix: Additional Empirical Results

B.1 Merging the CDS and 13-F datasets

We merge CDS trade data with 13-F filings using the names of the institution of each trader. Since the institutions' names do not match perfectly we approximate them using Levenshtein-edit distance. The Levenshtein edit distance is a measure of approximateness between strings: it is the total number of insertions, deletions and substitutions required to transform one string into another. We match the names with Levenshtein-edit distance less than 0.5. We only use the first three words of the identifiers when computing the Levenshtein edit distance because this is where the identifying parts of the institution's name tend to be.[26] We manually check the matched names to make sure there are no bad matches, which is feasible given there are not many institutions filing that also trade CDS, see Table 1.

In some cases, we find multiple DTCC account IDs, but only one CIK ID from EDGAR. That is, not everything is one-to-one between the data sets.[27] These cases happen because the DTCC IDs can be granular, while the 13-F filings tend to be more at the institution level. We keep the institution ID from the DTCC data, so one filing that is associated with a CIK in the EDGAR data, will be associated with multiple DTCC IDs in our data.

[26] After the first three words, there tend to be "filler" words such as "LTD" - "LIMITED", "CORP" or "CORPORATION".

[27] For example, the Edmond De Rothschild Holdings has an unique CIK in the EDGAR data, and it is associated with four DTCC IDs: EDMOND DE ROTHSCHILD EMERGING BONDS, EDMOND DE ROTHSCHILD Bond Allocation, EDMOND DE ROTHSCHILD QUADRIM 8 and EDMOND DE ROTHSCHILD QUADRIM 4.

B.2 Filing around the 45 day limit

Table 6 reports the results of (19) where we define the independent variable $F_{j,t-x}$ to be a dummy variable equal to one if institution j filed a 13-F report in the x-weeks previous to week t and that report was made between 42 and 48 days from the beginning of the quarter, a symmetric window around the cutoff.[28]

We find that narrowing the test of a 13-F to only those around the deadline strengthens our baseline results, suggesting that some degree of endogeneity for short delays. Whether looking at windows of one or two weeks and regardless of controlling for trade in the time period just before a report, we find a positive and significant impact of a 13-F filing on trade with periphery institutions and no effect on trade with core institutions. The coefficient estimates increase, nearly doubling in most specifications. For instance, if we compare column (1) in Table 6 to column (4) in Table 2, we find that narrowing the focus to 13-F reports around the deadline increases the impact of a 13-F report on trade with the periphery from 13.8 percentage points to 21.4 percentage points. Similarly the differential effect on trade with the periphery relative to the core increases from 14.8 percentage points to 25.8 percentage points. As much as these regressions control for endogeneity in filing delay, we see these results as confirming our theory.

B.3 Trading activity around the filing date

In Table 7, we add an additional regressor to (19) that controls for trade in the time period *just prior* to a 13-F filing,

$$Y_{ijt} = \beta_1 \frac{F_{j,t-x}}{Frequency} + \beta_2 \frac{F_{j,t+x}}{Frequency} + \text{Fixed Effects}_{jit} + \epsilon_{jit}, \tag{47}$$

where, as before, $F_{j,t-x}$ is a dummy equal to one if institution j filed a 13-F report in the x-weeks previous to week t and $F_{j,t+x}$ is a dummy equal to one if institution j filed a 13-F report in the x weeks following week t. The coefficient β_2 should identify trade activity immediately before a report. For instance, if front-running is a concern of 13-F filers that trade CDS indexes, then $\beta_2 > 0$ to indicate that institutions trade relatively higher immediately before reporting. Then, as suggested by Proposition 10, we should find an increase in the probability of trade in the time period after relative to the time period before a report, or $\beta_1 > \beta_2$.

We focus on the sample of filers trading US CDS indexes and report results using the two sets of fixed effects from above. Columns (1) and (2) of Table 7 look at the probability of trade in the week before versus the week after the week of the 13-F filing. Similarly, columns (3) and (4) broaden the horizon to two weeks before and after.

[28]We extend to 48 days because sometimes the deadline falls on a weekend or holiday. In these cases, the SEC extends the deadline to the first business day after 45 days past the beginning of the quarter. Likewise, we include the 42 days as, if the 45-day deadline falls on a Monday, an institution may file on Friday of the prior week.

Table 6: Impact of a 13-F filing on trade (42-48 day filing delay).

	$x = 1$ week		$x = 2$ weeks	
	(1)	(2)	(3)	(4)
	Dependent Variable: Trade with Periphery, β^p			
Filed in week $t - x$, $F_{i,t-x}$	0.214**	0.216**	0.281***	0.281***
	(0.106)	(0.107)	(0.080)	(0.080)
Filed in week $t + x$, $F_{i,t+x}$		0.017		0.005
		(0.107)		(0.081)
R-squared	0.198	0.198	0.198	0.198
	Dependent Variable: Trade with Core, β^c			
Filed in week $t - x$, $F_{i,t-x}$	-0.043	-0.050	0.008	0.003
	(0.077)	(0.077)	(0.057)	(0.058)
Filed in week $t + x$, $F_{i,t+x}$		-0.092		-0.027
		(0.077)		(0.058)
R-squared	0.204	0.204	0.204	0.204
	Dependent Variable: Difference, $\beta^p - \beta^c$			
Filed in week $t - x$, $F_{i,t-x}$	0.256**	0.266**	0.273***	0.278***
	(0.104)	(0.104)	(0.078)	(0.079)
Filed in week $t + x$, $F_{i,t+x}$		0.107		0.032
		(0.104)		(0.079)
R-squared	0.119	0.119	0.119	0.119
Fixed Effects				
Week − index	yes	yes	yes	yes
Institution − quarter	yes	yes	yes	yes
Observations	460,512	458,712	458,640	455,040

Sample includes trades of US credit default swap indexes by regulated institutions or those trading CDS indexes on regulated institutions, that filed a 13-F report at least once in the sample period, 2013Q1-2017Q4. The independent variables, $F_{j,t-x}/Frequency$ and $F_{j,t+x}/Frequency$, are normalized dummies, where the dummies are equal to one if institution j filed a 13-F within the previous x weeks and within the following x weeks, respectively, to week t, conditional on filing near the filing deadline defined as 42 to 48 days past the beginning of the quarter. The two dependent variables are dummies if institution j traded CDS index i in week t with a periphery and core institution, respectively. Test on difference: tests whether the difference in the coefficients is equal to zero. Standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Table 7: Impact of 13-F filing on trade in week(s) after report relative to before.

	$x = 1$ week		$x = 2$ weeks	
	(1)	(2)	(3)	(4)
	Dependent Variable: Trade with Periphery, β^p			
Filed in week $t - x$, $F_{i,t-x}$	0.237***	0.140*	0.271***	0.149**
	(0.075)	(0.074)	(0.059)	(0.060)
Filed in week $t + x$, $F_{i,t+x}$	0.124*	0.027	0.124**	0.018
	(0.075)	(0.074)	(0.059)	(0.060)
Prob>F: $\beta_1 = \beta_2$	0.266	0.259	0.058	0.087
R-squared	0.176	0.198	0.176	0.198
	Dependent Variable: Trade with Core, β^c			
Filed in week $t - x$, $F_{i,t-x}$	0.102*	-0.016	0.137***	-0.018
	(0.054)	(0.054)	(0.042)	(0.043)
Filed in week $t + x$, $F_{i,t+x}$	0.053	-0.065	0.130***	-0.022
	(0.054)	(0.054)	(0.043)	(0.043)
Prob>F: $\beta_1 = \beta_2$	0.499	0.497	0.891	0.940
R-squared	0.183	0.204	0.183	0.204
	Dependent Variable: Difference, $\beta^p - \beta^c$			
Filed in week $t - x$, $F_{i,t-x}$	0.135*	0.156**	0.133**	0.167***
	(0.073)	(0.073)	(0.058)	(0.058)
Filed in week $t + x$, $F_{i,t+x}$	0.072	0.092	-0.005	0.040
	(0.073)	(0.073)	(0.058)	(0.058)
Prob>F: $\beta_1 = \beta_2$	0.522	0.516	0.066	0.091
R-squared	0.097	0.119	0.097	0.119
Fixed Effects				
Week $-$ index	yes	yes	yes	yes
Institution	yes	no	yes	no
Institution $-$ quarter	no	yes	no	yes
Observations	458,640	458,640	454,896	454,896

Notes: Sample includes trades of US credit default swap indexes by regulated institutions or those trading CDS indexes on regulated institutions, that filed a 13-F report at least once in the sample period, 2013Q1-2017Q4. The independent variables, $F_{j,t-x}/Frequency$ and $F_{j,t+x}/Frequency$, are normalized dummies, where the dummies are equal to one if institution j filed a 13-F within the previous x weeks and within the following x weeks, respectively, to week t. The two dependent variables are dummies if institution j traded CDS index i in week t with a periphery and core institution, respectively. Test on difference: tests whether the difference in the coefficients is equal to zero. Standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

As in Table 2, the first row of the table shows that the probability of trade with periphery institutions increases in the week (two weeks) after a filing. In the week (two weeks) prior to the filing, the probability of trade with non-central institutions is also slightly above the average, but importantly the magnitude is always smaller than in the time period after. In fact, when we control for institution-quarter fixed effects, we find no statistical effect on trade with periphery institutions *just before a report*. However, we find a report increases the probability of trade with non-central institutions by around 14-15% in the week to two weeks *just after a report*.

B.4 Can a model of signaling explain the observed trade outcomes?

Consider a model where, as in Duffie et al. (2005), the asset supply is exogenous, investors meet randomly, and the investors' type is observable by trade counterparties in a meeting. A fraction of investors have valuation ν_l and a meet a trade counterparty at rate λ_H, while the rest of the investors have valuation $\nu_t \in \{\nu_l, \nu_h\}$, with $\nu_l < \nu_h$ and $\Pr[\nu_t = \nu_l] = \Pr[\nu_t = \nu_h] = 1/2$ for all t, and meet a trade counterparty at rate $\lambda_L < \lambda_H$. That is, there are three types of investors in this economy: hL investors (high valuation, low speed), lL investors (low valuation, low speed), and lH investors (low valuation, high speed).

hL investors buy from lL and lH investors –hL investors are buyers. lL investors sell to lH and hL investors –lL investors are sellers. lH investors buy from lL investors and sell to hL investors –lH investors are intermediaries. Because $\lambda_L < \lambda_H$, lL and hL investors populate the periphery, while lH investors populate the core (Üslü, 2019; Farboodi et al., 2017). The assumption that the lH investors have persistent valuation ν_l imply that the identity of the core institutions is not changing through time. This is not only consistent with the way we want to interpret core institutions in our model but, more importantly, consistent with the data, where the identity of the institutions populating the core is highly persistent.

In our data, almost all institutions that file Form 13-F are periphery institutions in CDS trading markets.[29] Thus, consider now the experiment where an investor populating the periphery (lL or hL, at a given point in time) must file Form 13-F within a window of time $[T_0, T_0 + T]$, where $T_0 + \tilde{t}$ is the filing date, $\tilde{t}$ denoting the delay chosen by the investor. Further, assume that, when the investor files Form 13-F, other investors can use semi-direct search towards the filing investor: when searching for this investor, they search for the filing investor at rate λ_H. Why would investors search for the investor that filed form 13-F? In our theory, filing does not affect matching probabilities, but affects trade outcomes and terms of trade. Because of complete information, this cannot happen in the model we are discussing here. Rather, the act of filing can be used by the 13-F investor to convey information to potential trade counterparties prior to meeting, and thus induce them to search for her. That is, the 13-F investor can delay filings in such a particular way to signal her trade interests to the market. In this model, dispersion in the observed distribution of delays is the result of optimal signaling.

There are four possible signaling outcomes: (i) the investor only files when she is hL and she is not holding an asset, signaling that she is a buyer, (ii) the investor only files when she is lL and she is holding an asset, signaling that she is a seller, (iii) the investor files when she is hL and she is not holding an asset, or when she is lL and she is holding an asset, signaling that she is mismatched and, (iv) the investor files at random times, independently of her valuation and holding status.

[29] In the data, 99.7% of filings are done by non top-5 institutions.

The objective of signaling is two-fold. First, signaling increases the probability of matching, as it induces other investors to direct their search towards the 13-F investor. Second, conditional on a meeting, signaling increases the probability of a successful match. In case (i), only lL and lH investors that are holding an asset search for the 13-F investor. In case (ii), only lH and hL investors not holding an asset search for for the 13-F investor. And in case (iii) the 13-F investor avoids potential meetings with lL investors not holding an asset, and hL investors.

Implications and the baseline regression. When the 13-F investor uses the filing event to signal her type, she is choosing $\tilde{t}$ strategically. Thus, this model has the potential to explain the heterogeneity in delays observed in the data using an endogenous mechanism. Also, as a result of signaling, trade probability goes up after filing, consistent with the finding that trade probability goes up when trading with both core and periphery investors (Table 2). Further, the fact that, when searching for the filing investor, all other investors search at rate λ_h implies that, relative to before filing, the trade probability increases more with periphery than core investors. As a result, the simple model presented here is potentially able to reproduce our baseline regressions.

Other tests to disentangle the two theories. In the Buyer's signaling case, we should observe a pronounced increase in buying activity by 13-F investors after filing. In the Seller's signaling case, we should observe a pronounced increase in selling activity by 13-F investors after filing. In the mismatched case, we should observe an increase in both buying and selling activity after filing. Table 8 presents trade activity statistics around filing dates, for those institutions that file Form 13-F. Column 1 shows that around half of trades that occur in a window of 2 weeks (starting one week before filing, and ending one week after filing) involve the investor buying an asset, while the other half involve the investor selling an asset.[30] Conditional on buying an asset, 56% of trades occur after filing and, conditional on selling an asset, 51% of trades occur after filing. The fact that we see a considerable amount of buying and selling activity before and after filing suggests that the the Buyer's and Seller's signaling cases do not explain why institutions delay their filings: institutions should defer and buy or sell after filing. Our model, and the mismatch signaling equilibrium in the simple model, are both consistent with large buying and selling activity before and after filing events.

Related to the last exercise, we can repeat our baseline regressions, but for buyers and sellers independently. If we were to find that filing Form 13-F affects the trade probability only of buyers or sellers, but not of both, then that would provide evidence consistent with the seller's signaling equilibrium, or the buyer's signaling equilibrium. In Table 9, we report the results of the baseline regressions, (19), where we now define the dependent variables $D_{ijt}^{c/p}$ as dummies equal to one if institution j traded CDS index i in week t *a*s a buyer or seller, respectively. We are interested if the effect of trading CDS is active both on the buy and sell side of the market, or is dominated by one side.

[30]Column 2 in Table 8 shows that the results are robust to using a window of 4 weeks, centered at the filing date.

Table 8: Buying and selling activity for filers around filing events

	1-week	2-weeks
All trades - full window (before & after)	5,487	11,067
Trades in window after filing	2,991	5,758
Trades in window before filing	2,496	5,309
Trades when buying - full window (before & after)	2,773	5,624
Trades in window after filing	1,565	3,001
Trades in window before filing	1,208	2,623
Trades when selling - full window (before & after)	2,715	5,530
Trades in window after filing	1,426	2,757
Trades in window before filing	1,289	2773

Notes: Sample includes trades by investors filing form 13-F that trade US indexes. 1-week: For a given filing event, defines a one-week window of trades, either before or after filing. In this case, a full window has two weeks, one week before filing, and one week after filing. 2-weeks: For a given filing event, defines a two-week window of trades, either before or after filing. In this case, a full window has four weeks, two weeks before filing, and two weeks after filing.

We see that the effect of a 13-F report on trade with the periphery is positive and similar whether on the buy- or sell-side of the market. The effect of a 13-F on trade with the core is always below the effect with the periphery, although splitting the sample we lose power so the standard errors increase. The fact that the effect of filing affects the trade probability of both buyers and sellers and that it affects differentially their trading activity with core and periphery investors is inconsistent with the the buyer and seller's signaling equilibria in the simple model, while in line with what is expected from our model and the mismatched signaling equilibrium.[31]

The last test exploits the result that in any signaling equilibrium, a delay $\tilde{t} < T$. To see why this is the case, consider the scenario where the investor reaches time $T_0 + T$ and still needs to file. Because the investor must file, there is no information content in the act of filing with delay $\tilde{t} = T$. As a result, in any signaling equilibrium, $\tilde{t} < T$. The fact that a large fraction of filing events involve maximum delays (as previously discussed, 66% of filings exhibit a delay of 42 to 48 days) is indicative of the inability of the signaling model to account for most of the variation in the data. Further, Table 6. There, we showed that our baseline results hold with

[31]The proof of Proposition 10 suggests that our results should hold *for at least one side of the market*, but not necessarily both depending on the weights ξ_o and ξ_n. For instance if $\xi_o = 1$, then we should only see the effects of a 13-F report on the buyer-side of the market. The intuition is if sellers always make the offer, or $\xi_o = 1$, then an institution's private information is only valuable when they trade with a seller, as a buyer. The opposite is true when $\xi_n = 1$.

Table 9: Impact of a 13-F filing on trade, buy-side vs. sell-side

	Buy-side		Sell-side	
	(1)	(2)	(3)	(4)
Trade with Periphery, $\frac{\beta^p}{Freq^p}$	0.232***	0.146*	0.244**	0.150*
	(0.088)	(0.087)	(0.089)	(0.089)
R-squared	0.150	0.171	0.148	0.167
Trade with Core, $\frac{\beta^c}{Freq^c}$	0.097	-0.007	0.134**	0.025
	(0.067)	(0.066)	(0.065)	(0.065)
R-squared	0.145	0.163	0.144	0.162
Test on difference, $\frac{\beta^p}{Freq^p} - \frac{\beta^c}{Freq^c}$	0.135	0.153*	0.110	0.125
	(0.089)	(0.089)	(0.089)	(0.089)
Fixed Effects				
$Week - index$	yes	yes	yes	yes
Institution	yes	no	yes	no
$Institution - quarter$	no	yes	no	yes
Observations	460,512	460,512	460,512	460,512

Sample includes trades of US credit default swap indexes by regulated institutions or those trading CDS indexes on regulated institutions, that filed a 13-F report at least once in the sample period, 2013Q1-2017Q4. The independent variable is a dummy equal to one if institution j filed a 13-F in the previous week. The two dependent variables are dummies if institution j traded CDS index i in week t as a buyer or seller with a periphery or core institution, respectively. We normalize the coefficients of each regression by the frequency of trading with each group so that coefficients are comparable. Test on difference: tests whether the difference in the normalized coefficients are equal to zero. Standard errors are in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

added strength for those filing events close to the filing limit. This is at odds with the all of the signaling equilibria in the simple model.

Overall, in this section we ran several tests aimed to study whether a theory of signaling, that explains the observed endogeneity in filing times that we observe in the data and our baseline regression, can rationalize all of our our empirical findings without the need of relying on the mechanism that we explore in our model that builds on private information about private values. The empirical results presented in this section provide added confidence to the relevance of our theory to explain the observed empirical regularities of OTC trading in CDS markets. In the end, what seems to be the crucial feature to explain the empirical regularities is the presence of private information about private values.

C Online Appendix - Not For Publication

In this online appendix we collect theoretical and empirical results of the paper 'An Information-based theory of financial intermediation'.

C.1 An alternative trade mechanism

In this section, we explore a mechanism that maximizes the expected trade surplus in the bilateral meeting, as in Myerson and Satterthwaite (1983). The mechanism we currently use—ask and bid prices that maximize the owner's and non-owner's expected surplus, respectively—does not maximize trade surplus for two reasons. The first reason is the following. Consider a meeting where the owner designs the trade mechanism, but she does not observe the type of the non-owner, while the non-owner does observe the type of the owner. In this case, the owner will distort trade in order to maximize her trade profits, as described in Corollary 1. However, if the non-owner would have been chosen to design the mechanism, trade surplus would have been maximized, as the non-owner observes the type of the owner.

The second reason the mechanism we use does not maximize trade surplus relates to the incentives faced by investors. In a meeting where both owner and non-owner do not observe the type of their counterparty, Myerson and Satterthwaite (1983) show that there is no mechanism that implements the ex-post efficient allocation. When either owner or non-owner is chosen to design the trade mechanism—the owner with probability ξ_o or the non-owner with probability ξ_n—they are both willing to give up total surplus in order to maximize their individual surplus.

Whenever owner or non-owner observes the type of their trade counterparty, ex-post efficiency can be achieved by assigning all the gains from trade to the informed party. When neither side is informed, the trade mechanism used is the one in Myerson and Satterthwaite (1983). We show our main results remain unchanged using this alternative mechanism.[32]

C.1.1 Bilateral trade

We consider a mechanism that maximizes the total gains from trade, or trade surplus, in a meeting. In any meeting, there are four possible information structures:

1. both owner and non-owner know each other types;
2. the owner knows the non-owner's type and the non-owner does not know the owner's type;
3. the owner does not know the the non-owner's type and the non-owner knows the owner's type; and

[32]All proofs in this section are variations of previous proofs or directly derived from Myerson and Satterthwaite (1983), so we omit these proofs here. They are available upon request.

4. neither owner or non-owner know each other types.

In case i, we can directly apply Nash bargaining since we have complete information. In this case, we use the same notation as before to represent the bargaining power of investors, with ξ_o denoting the owner's bargaining power and $\xi_n = 1 - \xi_o$ denotes the non-owner's bargaining power. In case ii, to maximize the total surplus in the trade the mechanism just gives all the bargain power to the owner. Since the owner knows the type of the non-owner, she will sell the asset whenever the reservation value of the non-owner is above her own and will extract all the surplus. In a similar way, in case iii, the mechanism just gives all the bargain power to the non-owner. Since the non-owner knows the type of the owner, she will buy the asset whenever the reservation value of the owner is below her own and will extract all the surplus in the trade.

In case iv, we have two-sided incomplete information so we apply the mechanism propose by Myerson and Satterthwaite (1983), which maximizes the expected gains from trade in a meeting. In order to characterize the outcomes from such mechanism, it is without loss of generality to focus on direct mechanisms due to the revelation principle. A direct mechanism is a pair of functions $m = (p, x) : [\underline{\Delta}, \bar{\Delta}] \times [\underline{\Delta}, \bar{\Delta}] \to [0, 1] \times \mathbb{R}$, where, for given reservation values Δ_o and Δ_n of owner and non-owner, $p(\Delta_o, \Delta_n)$ is the probability of transferring the asset from the owner to the non-owner, and $x(\Delta_o, \Delta_n)$ is the transfer from the non-owner to the owner. The mechanisms are also going to be a function of the screening expertise α_o and α_n in equilibrium, but we omit this argument here to keep the notation short.

Let $M_o(\cdot; \alpha_o)$ and $M_n(\cdot; \alpha_n)$ be the cumulative distribution of reservation values of owner and non-owner conditional on α_o and α_n, and $m_o(\cdot; \alpha_o)$ and $m_n(\cdot; \alpha_n)$ the respective densities. As before, we omit the argument α_o and α_n from the distributions above to keep the notation short. The mechanism that maximizes the expected gains from trade in the meeting solves

$$\max_{m} \int \int p(\Delta_o, \Delta_n) \left[\Delta_n - \Delta_o\right] dM_o(\Delta_o) dM_n(\Delta_n) \tag{48}$$

subject to

$$IR_o : \quad x_o(\Delta_o) - p_o(\Delta_o)\Delta_o \geq 0; \tag{49}$$

$$IC_o : \quad x_o(\Delta_o) - p_o(\Delta_o)\Delta_o \geq x_o(\hat{\Delta}_o) - p_o(\hat{\Delta}_o)\Delta_o; \tag{50}$$

$$IR_n : \quad p_n(\Delta_n)\Delta_n - x_n(\Delta_n) \geq 0 \text{ and} \tag{51}$$

$$IC_n : \quad p_n(\Delta_n)\Delta_n - x_n(\Delta_n) \geq p_n(\hat{\Delta}_n)\Delta_n - x_n(\hat{\Delta}_n); \tag{52}$$

where

$$x_o(\Delta_o) = \int x(\Delta_o, \Delta_n) m_n(\Delta_n) d\Delta_n, \quad p_o(\Delta_o) = \int p(\Delta_o, \Delta_n) m_n(\Delta_n) d\Delta_n,$$
$$x_n(\Delta_n) = \int x(\Delta_o, \Delta_n) m_o(\Delta_o) d\Delta_o \quad \text{and} \quad p_n(\Delta_n) = \int p(\Delta_o, \Delta_n) m_o(\Delta_o) d\Delta_o.$$

Equations (49) and (51) are the usual individual rationality constraints. They guarantee that the mechanism generates enough incentives for both agents to participate. Equations (50) and (52) are the usual incentive compatibility constraints. They guarantee that the mechanism generates enough incentives for both agents to truthfully reveal their reservation values.

C.1.2 Expected gains from trade

The expected gains from trade of a type θ_o owner in a meeting is

$$\pi_o(\theta_o) = \int \Big\{ \alpha_o \left(1 - \alpha_n + {}^{\alpha_n}/2\right) \left[\max(\Delta_o, \Delta_n) - \Delta_o\right] \\ + (1-\alpha_o)(1-\alpha_n)\left[x(\Delta_o,\Delta_n) - p(\Delta_o,\Delta_n)\Delta_o\right] \Big\} d\frac{\Phi_n(\theta_n)}{1-s}, \tag{53}$$

and the expected gains from trade of a type θ_n non-owner in a meeting is

$$\pi_n(\theta_n) = \int \Big\{ \alpha_n \left(1 - \alpha_o + {}^{\alpha_o}/2\right) \left[\max(\Delta_o, \Delta_n) - \Delta_o\right] \\ + (1-\alpha_n)(1-\alpha_o)\left[p(\Delta_o,\Delta_n)\Delta_n - x(\Delta_o,\Delta_n)\right] \Big\} d\frac{\Phi_o(\theta_o)}{s}. \tag{54}$$

The expected gains from trade described in (53) and (54) are analogous to the ones described in (53) and (54). The difference is how investors split the trade surplus. Here, trade occurs according to the arrangement discussed in subsection C.1.1, where transfers are designed to maximize expected surplus.

C.1.3 Value functions and reservation value

In this section we describe the value functions for owners and non-owners and we provide an expression for the reservation value Δ. These objects are analogous to the ones derived in subsection 3.3. The difference here is that we use the expected gains from trade of owners and non-owners, π_o and π_o, that we computed in subsection C.1.2 instead of the one in subsection 3.2. The value function for an owner of a type θ is given by

$$rV_o(\theta) = \nu - \mu\left[V_o(\theta) - V_n(\theta)\right] + \lambda(1-s)\pi_o(\theta). \tag{55}$$

Likewise, the value function for a non-owner of type θ is,

$$rV_n(\theta) = \eta\left[\max\{V_o(\theta), V_n(\theta)\} - V_n(\theta)\right] + \lambda s \pi_n(\theta). \tag{56}$$

Using equations (55)-(56), we can compute the reservation value function for an investor of type θ, $\Delta(\theta) \equiv V_o(\theta) - V_n(\theta)$. The reservation value $\Delta(\theta)$ solves

$$r\Delta(\theta) = \nu - \mu\Delta(\theta) - \eta\max\{\Delta(\theta), 0\} + \lambda(1-s)\pi_o(\theta) - \lambda s\pi_n(\theta). \tag{57}$$

C.1.4 The distribution of assets

The change over time in the density of owners with type θ is

$$\dot{\phi}_o(\theta) = \eta\phi_n(\theta)\mathbb{1}_{\{\Delta(\theta)\geq 0\}} - \mu\phi_o(\theta) - \lambda\phi_o(\theta)\bar{q}_o(\theta) + \lambda\phi_n(\theta)\bar{q}_n(\theta), \tag{58}$$

where

$$q(\theta_o, \theta_n) = [1 - (1-\alpha_o)(1-\alpha_n)]\mathbb{1}_{\{\Delta_n \geq \Delta_o\}} + (1-\alpha_o)(1-\alpha_n)p(\Delta_o, \Delta_n) \tag{59}$$

is the probability of trade between a type θ_o owner and a type θ_n non-owner,

$$\bar{q}_o(\theta) = \int q(\theta, \theta_n)\phi_n(\theta_n)d\theta_n \tag{60}$$

is the probability that a type θ owner sells an asset in a meeting, and

$$\bar{q}_n(\theta) = \int q(\theta_o, \theta)\phi_o(\theta_o)d\theta_o \tag{61}$$

is the probability that a type θ non-owner buys an asset in a meeting. The difference between the law of motion in (12) and (58) comes from the use the Myerson and Satterthwaite (1983) trade mechanism. We can see this from the equations for $q(\Delta_o, \Delta_n)$ in the two different settings.

As in subsection 3.4, we can obtain an expression for the density of non-owners of type θ from the equilibrium condition

$$\phi_o(\theta) + \phi_n(\theta) = f(\theta), \tag{62}$$

and an expression for total asset supply is given by

$$s = \int \phi_o(\theta)d\theta. \tag{63}$$

C.1.5 Equilibrium

We focus on symmetric steady-state equilibrium.

Definition 2. *A family of direct mechanism, reservation values and distributions, $\{m = (p, x), \Delta, \phi_o, \phi_n, s\}$, constitutes a symmetric steady-state equilibrium if it satisfies:*

1. *the mechanism $m = (p, x)$ solves problem (48);*

2. *the reservation value of investors $\Delta(\cdot)$ is continuous and satisfies (57), where π_o and π_n are given by (53) and (54); and*

3. *the density of owners ϕ_o satisfies (58) with $\dot{\phi}_o = 0$, the measure of non-owners ϕ_n satisfies (62), and the stock of assets s satisfies (63).*

As in section 4, the equilibrium definition does not include the value functions V_o and V_n because we can recover them from (55) and (56).

Proposition 13. *There exists a symmetric steady-state equilibrium.*

C.1.6 Intermediation

The trade protocol used here differs from the one used in Section 5, but our results regarding trading speed and centrality are the same.

Efficient ex-post trade in a bilateral meeting means that the buyer acquires the asset whenever her reservation value is above the reservation value of the seller. That is, if trade is ex-post efficient, then the probability of trade is $\mathbb{1}_{\{\Delta_n \geq \Delta_o\}}$. The Myerson and Satterthwaite (1983) implies that, under private information,[33] efficient ex-post trade cannot be achieved.

Proposition 14. *Consider a symmetric steady-state equilibrium* $\{m = (p, x), \Delta, \phi_o, \phi_n, s\}$. *Then, efficient ex-post trade in the bilateral meetings is not achieved. That is,* $p(\Delta_o, \Delta_n) < \mathbb{1}_{\{\Delta_n \geq \Delta_o\}}$ *for a positive measure of* Δ_o *and* Δ_n. *Moreover,*

- $\int p(\Delta_o, \Delta_n) dM_n < \int \mathbb{1}_{\{\Delta_n \geq \Delta_o\}} dM_n$, *and*
- $\int p(\Delta_o, \Delta_n) dM_o \leq \int \mathbb{1}_{\{\Delta_n \geq \Delta_o\}} dM_o$, *with strict inequality if* $\Delta_n > 0$.

The probability of trade between a type θ_o owner and a type θ_n non-owner is

$$q(\theta_o, \theta_n) = [1 - (1 - \alpha_o)(1 - \alpha_n)]\mathbb{1}_{\{\Delta_n \geq \Delta_o\}} + (1 - \alpha_o)(1 - \alpha_n) p(\Delta_o, \Delta_n).$$

Since α_o and α_n are smaller than one with positive probability, Proposition 14 implies that $q(\theta_o, \theta_n)$ is smaller than one for a positive measure of θ_o and θ_n. Moreover, keeping the reservation value constant, we have that

$$\frac{q(\theta_o, \theta_n)}{\partial \alpha_o}\Big|_{\Delta(\theta_o) = \bar{\Delta}} = (1 - \alpha_n) \left[\mathbb{1}_{\{\Delta_n \geq \Delta_o\}} - p(\Delta_o, \Delta_n)\right].$$

In a similar way,

$$\frac{q(\theta_o, \theta_n)}{\partial \alpha_n}\Big|_{\Delta(\theta_n) = \bar{\Delta}} = (1 - \alpha_o) \left[\mathbb{1}_{\{\Delta_n \geq \Delta_o\}} - p(\Delta_o, \Delta_n)\right].$$

This brings us to our next result.

Proposition 15. *Consider a symmetric steady-state equilibrium* $\{m = (p, x), \Delta, \phi_o, \phi_n, s\}$, *and let the types* $\theta = (\alpha, \nu)$ *and* $\hat{\theta} = (\hat{\alpha}, \hat{\nu})$ *satisfy* $\Delta(\theta) = \Delta(\hat{\theta})$ *and* $\alpha > \hat{\alpha}$. *Then,*

- $\bar{q}_o(\theta) > \bar{q}_o(\hat{\theta})$, *and*
- $\bar{q}_n(\theta) \geq \bar{q}_n(\hat{\theta})$, *with strict inequality if* $\Delta(\theta) = \Delta(\hat{\theta}) > 0$.

Proposition 15 is intuitive. We know from Proposition 14 that trade is distorted in meetings under private information—that is, $p(\Delta_o, \Delta_n) < \mathbb{1}_{\{\Delta_n \geq \Delta_o\}}$ for a positive measure of Δ_o and Δ_n. Since investors with higher screening expertise are less likely to be in those meetings, they are less likely to have their trades distorted.

From Proposition 15 we can derive our main centrality result below.

[33] To be more specific, without common knowledge of gains from trade and connected support for valuations.

Proposition 16. *Consider a symmetric steady-state equilibrium* $\{m = (p, x), \Delta, \phi_o, \phi_n, s\}$*, and let the types* $\theta = (\alpha, \nu)$ *and* $\hat{\theta} = (\hat{\alpha}, \hat{\nu})$ *satisfy* $\Delta(\theta) = \Delta(\hat{\theta})$ *and* $\alpha > \hat{\alpha}$*. Then,*

- *if* $\Delta(\theta) = \Delta(\hat{\theta}) < 0$*, we have that* $c(\theta) = c(\hat{\theta}) = 0$*, and*
- *if* $\Delta(\theta) = \Delta(\hat{\theta}) \geq 0$*, we have that* $c(\theta) > c(\hat{\theta}) > 0$*.*

Moreover, if an investor type $\theta^* = (\alpha^*, \nu^*)$ *is the most central, then* $\alpha^* = 1$ *and* $c(\theta^*) > c(\theta)$ *for all* $\theta \in \Theta$ *satisfying* $\alpha < 1$*.*